Divinycell®

SANDWICH CONCEPT

SANDWICH

Whenever new materials or production methods appear there is a resistance to use them. Mostly the resistance originates in conservatism and ignorance. The only way to overcome the resistance is to try to teach and convince the opponents.

This handbook has been written to spread knowledge about sandwich and understanding of its behaviour.

HISTORY

Historically, the principal of using two cooperating faces with a distance between them was introduced by Delau about 1820. The first extensive use of sandwich panels was during World War II. In the ”Mosquito” aircraft sandwich was used, mainly because of the shortage of other materials in England during the war. The faces were made of veneer and the core of balsa wood.

During World War II the first theoretical writings about sandwich appeared. In the 50’s the development was mainly concentrated on honeycomb materials. Honeycomb was mainly used as core material in the aircraft industry. However, it had some limitations, for example there were big problems with corrosion.

At the end of the 50’s and during the 60’s different cellular plastics were produced, suitable as core materials. In the beginning rather soft materials were used because of their insulation properties, for example polystyrene and polyurethane.

Later it was possible to produce harder cellular plastics with higher densities and by that time sandwich became a very useful and flexible concept. Today there is an enormous number of different qualities of cellular plastics as core materials.

MAIN PRINCIPLES

Sandwich is built up of three elements, see Fig. 1.1.

* two faces
* core
* joints

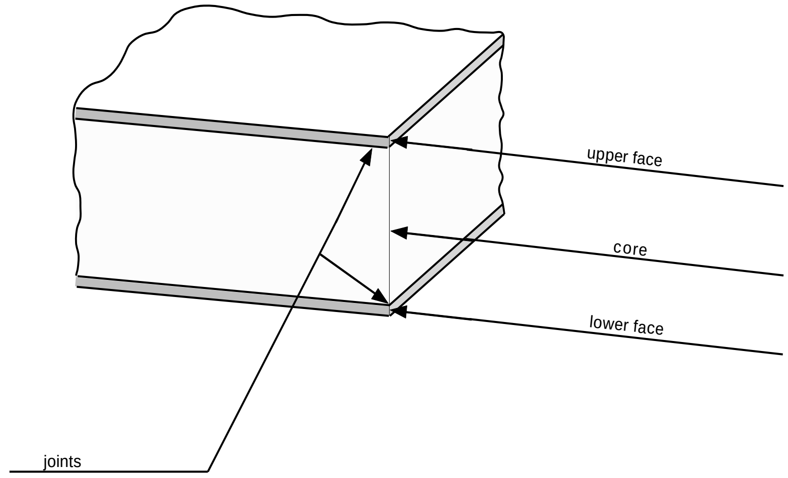


Fig. 1.1 Sandwich

Every part has its specific function to make it work as a unit.

The aim is to use the material with a maximum of efficiency. The two faces are placed at a distance from each other to increase the moment of inertia, and thereby the flexural rigidity, about the neutral axis of the structure. A comparison could be made with a solid beam. A Sandwich beam of the same width and weight as a solid beam has a remarkably higher stiffness because of its higher moment of inertia.

Fig. 1.2 demonstrates, as a simple example, the difference in flexural rigidity for a solid beam versus a sandwich beam.

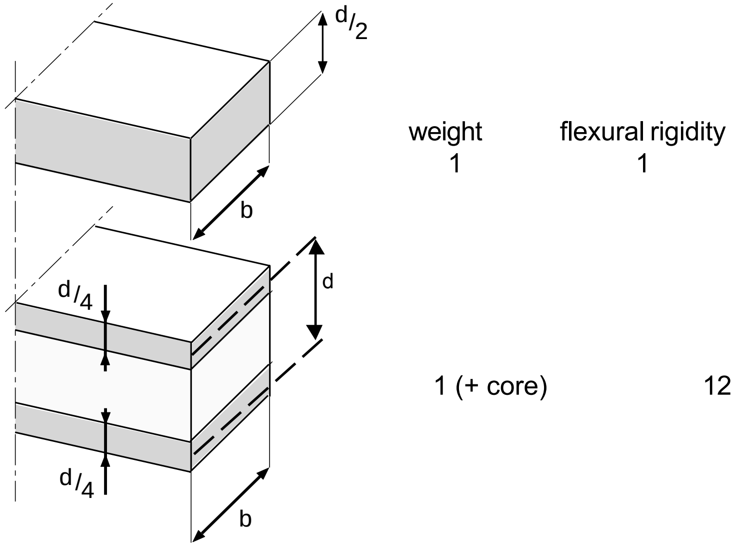


Fig. 1.2 Comparison of stiffness in bending between solid beam and sandwich beam

An important difference in comparing an I-beam with a sandwich beam is the possibility for each to bear transverse loads. For an I–beam the web is stiff enough to give Navier's assumption validity, (i.e. plane cross sections remain plane). In a sandwich beam the core material is usually not rigid in shear and the assumption is not fullfilled. In bending the shear deflection in the core is not negligible in most cases. There is also shear deflection in the faces but this can be ignored.

The effect of shear rigidity in the core is shown in fig. 1.3.

The upper case shows an ideal sandwich beam which is relatively stiff in shear. It is obvious how the faces cooperate without sliding over each other.

The lower case shows, as a comparison, a sandwich beam which is not very rigid in shear. Here the faces do not cooperate and the faces work as plates in bending, independent of each other. The local flexural rigidities for the faces can in most cases be ignored. Accordingly, the result of a core that is weak in shear is a loss of the sandwich effect.

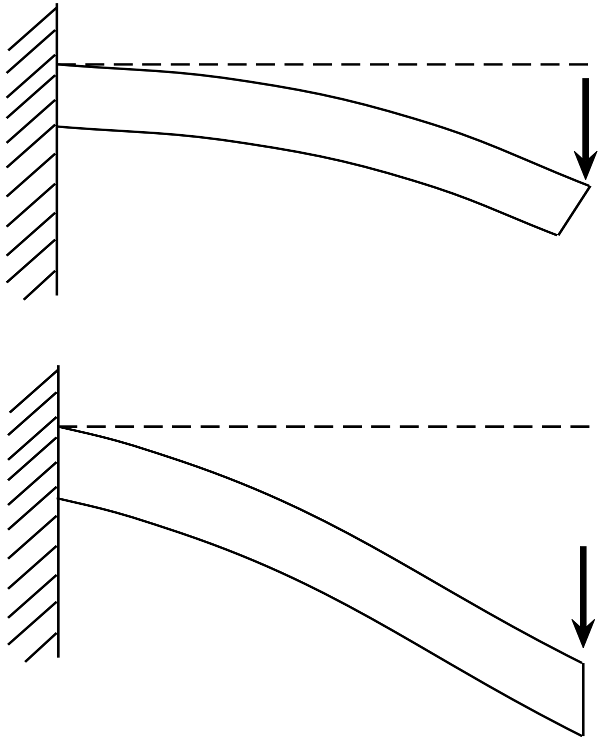


Fig 1.3 Comparison between cores that are rigid or weak in shear

Each of the parts in sandwich have their particular functions and will be described.

a/ The faces

The faces carry the tensile and compressive stresses in the sandwich. The local flexural rigidity is so small that it can often be ignored. Conventional materials such as steel, stainless steel and aluminium are often used for face material. In many cases it is also suitable to choose fibre- or glass- reinforced plastics as face materials. These materials are very easy to apply. Reinforced plastics can be tailored to fulfill a range of demands like anisotrophic mechanical properties, freedom of design, excellent surface finish etc.

Faces also carry local pressure. When the local pressure is high the faces should be dimensioned for the shear forces connected to it.

b/ The core

The core has several important functions. It has to be stiff enough to keep the distance between the faces constant. It must also be so rigid in shear that the faces do not slide over each other. The shear rigidity forces the faces to cooperate with each other. If the core is weak in shear the faces do not cooperate and the sandwich will lose its stiffness. (See fig. 1.3).

This presentation demonstrates that it is the sandwich structure as a whole that gives the positive effects. However, it should be mentioned that the core has to fulfill the most complex demands. Strength in different directions and low density are not the only properties that the core has to have. Often there are special demands for buckling, insulation, absorption of moisture, ageing resistence, etc.

c/ Adhesive (Bonding layer)

To keep the faces and the core co-operating with each other the adhesive between the faces and the core,must be able to transfer the shear forces between the faces and the core. The adhesive must be able to carry shear and tensile stresses. It is hard to specify the demands on the joints. A simple rule is that the adhesive should be able to take up the same shear stress as the core.

ASSUMPTIONS

In this chapter it is assumed that the faces are thin and of the same thickness. Shear and bending strains in the faces are small and can be ignored. The shear stress is assumed to be constant throughout the thickness of the core at any given section. For a beam with faces on the sides, the shear and bending strains in the side faces cannot be ignored.

In this chapter the beams are considered narrow. The conditions and directions for when a beam is to be considered narrow or wide are found in the chapter ”Beams considered narrow or wide”.

SIGN CONVENTION FOR BENDING OF BEAMS

The sign conventions to be adopted for deflection, slope, curvature, bending moment and shear forces are illustrated in fig 2.1.

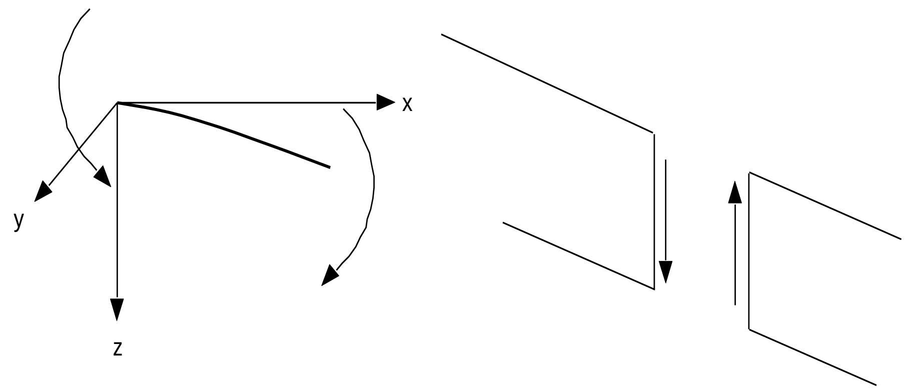


Fig. 2.1. Sign conventions. Left, positive deflection, slope and curvature; negative bending moment. Right, positive shear force, shear stress and shear strain.

Loads and deflections (w) are measured positive downwards, in the direction of the z-axis. As a result of the choice of sign convention it is necessary to introduce negative signs in some of the relationships between distributed load (q), shear force (Q), bending moment (M), slope (dw/dx), and deflection (w).

For reference, the full set of relationships, with the correct signs, is given:

$$\left.\begin{array}{ll} \text{Deflection} & w \\ \text{Slope} & + dw/dx = w' \\ \text{Curvature} & + w'' \\ - M & + Dw'' \\ - Q & + Dw''' \\ + q & + Dw^{(4)} \end{array}\right\} \qquad (2.1)$$

FLEXURAL RIGIDITY

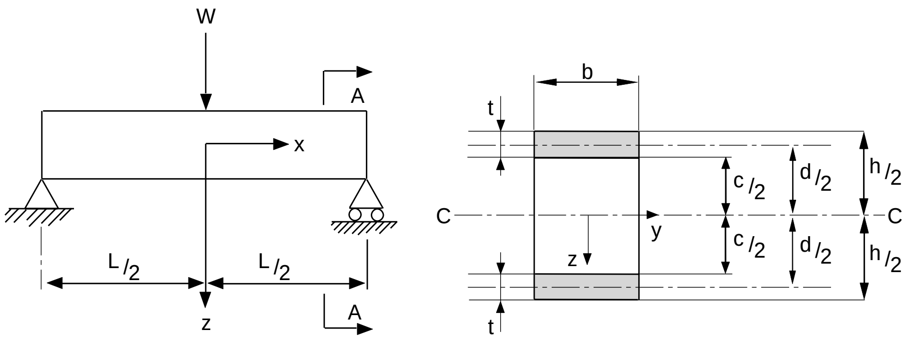
The theory for engineering stresses in beams is easily adapted to sandwich beams with some modifications. Effects caused by shear deflections in the core must be added and certain terms may be neglected when calculating flexural rigidity.

To use ordinary beam theory we should first find a simple way to calculate the flexural rigidity, here denoted D, of the beam. In an ordinary beam D would be the product of the modulus of elasticity, (E) and the second moment of area (I). In a sandwich beam D is the sum of the flexural rigidities of the different parts, measured about the centroidal axis of the entire section:

$$D = E_f \frac{bt^3}{6} + E_f \frac{btd^2}{2} + E_c \frac{bc^3}{12} \qquad (2.2)$$

E_f and E_c are the moduli of elasticity of the faces (index f) and the core (index c) respectively. Dimensions according to fig. 2.1.

Fig. 2.2. Dimensions of sandwich beam. Section AA on right.



The first term in equation (2.2) is local flexural rigidity of the faces about their own centroidal axes. The second term is the first term transposed for bending about the centroidal axis of the entire cross section. The third term is flexural rigidity of the core about its own centroidal axis, which is the same as for the entire cross section.

The first term amounts to less than 1% of the second when:

$$\frac{d}{t} > 5.77 \qquad (2.3)$$

At a ratio of d/t > 11.55 the proportion is less than 0.25% and since we have assumed that the faces are thin the first term can for the present be ignored.

The third term amounts to less than 1% of the second (and may consequently be ignored) when:

$$\frac{E_f}{E_c} \cdot \frac{td^2}{c^3} > 16.7 \qquad (2.4)$$

In many practical sandwich beams this condition is fulfilled but, considering the many combination possibilities of Divinycell, this term must be checked. The error may be too big to be acceptable. With condition (2.3) the expression for the flexural rigidity is:

$$D = E_f \frac{btd^2}{2} + E_c \frac{bc^3}{12} \qquad (2.5)$$

If condition (2.4) is fulfilled this expression will be reduced to:

$$D = E_f \frac{btd^2}{2} \qquad (2.6)$$

STRESSES

The stresses in a sandwich beam may also be determined by the use of theory for engineering stresses in beams, with a few modifications. Due to assumptions (sections remain plane and perpendicular to the centroidal axis) the strain at a point the distance z below the centroidal axis cc is Mz/D.

To obtain the bending stress at the same point the strain may be multiplied with the appropriate modulus of elasticity. For instance, the stresses in the faces and core are respectively:

$$\sigma_f = \frac{Mz}{D} E_f \left(\frac{c}{2} \le z \le \frac{h}{2}; -\frac{h}{2} \le z \le -\frac{c}{2} \right) \quad (2.7a)$$

$$\sigma_c = \frac{Mz}{D} E_c \left(-\frac{c}{2} \le z \le \frac{c}{2} \right) \quad (2.7b)$$

The maximum stresses are obtained with the maximum value of z within the interval. The ratio of the maximum membrane stress in the faces and the maximum core stress is $(E_f/E_c) \cdot (h/c)$.

The assumptions of the theory of bending lead to the common expression for the shear stress (τ) in a homogeneous beam at depth z, below the centroid of the cross section:

$$\tau = \frac{QS}{Ib} \quad (2.8)$$

Here Q is the shear force at the section under consideration, I is the second moment of area of the entire section about the centroid, b is the width at level z_1 and S is the first moment of area of the part of the section for which $z > z_1$. The familiar distribution of such shear stress in an I-beam is illustrated in Fig 2.3.

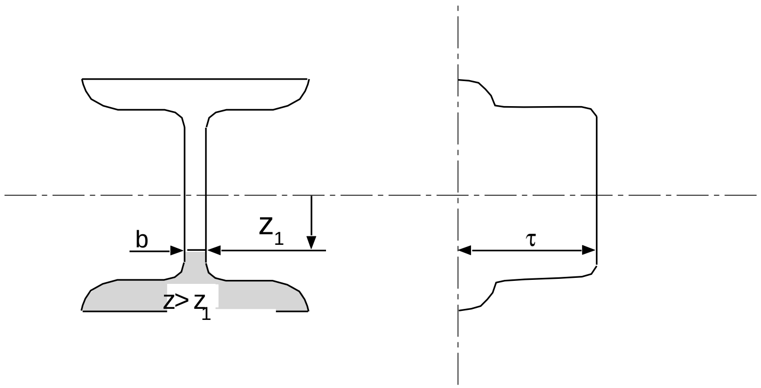


Fig. 2.3. Shear stress distribution in an I-beam.

For a sandwich beam, equation (2.8) must be modified to take into account the moduli of elasticity of the different elements of the cross section:

$$\tau = \frac{Q}{Db} \Sigma (SE) \quad (2.9)$$

In this expression D is the flexural rigidity of the entire section and Σ (SE) represents the sum of the products of S and E of all parts of the section for which $z < z_1$. For example, if equation (2.9) is used to determine the shear stress at a level z in the core of the sandwich in fig. 2.1,

$$\Sigma = (SE) = E_f \frac{btd}{2} + \frac{E_c b}{2} \left(\frac{c}{2} - z \right) \left(\frac{c}{2} + z \right)$$

The shear stress in the core is therefore

$$\tau = \frac{Q}{D} \left[E_f \frac{td}{2} + \frac{E_c}{2} \left(\frac{c^2}{4} - z^2 \right) \right] \quad (2.10)$$

An analogous expression may be obtained for the shear stress in the faces, and the complete shear stress distribution across the depth of the sandwich is illustrated in fig. 2.4a.The maximum shear stress in the core is obtained by inserting z = 0 in (2.10).

$$\tau = \frac{Q}{D} \left(E_f \frac{td}{2} + \frac{E_c}{2} \frac{c^2}{4} \right) \quad (2.11)$$

The ratio of the maximum core shear stress (at z = 0) to the minimum core shear stress (at z = ± c/2) is

$$\left(1 + \frac{E_c}{E_f} \frac{t}{4} \frac{c^2}{td} \right)$$

The second term amounts to less than 1% of the expression provided

$$4 \frac{E_f}{E_c} \frac{t}{c} \frac{d}{c} > 100 \quad (2.12)$$

If condition (2.12) is satisfied, the shear stress can be assumed constant over the thickness of the core. Because $d \approx c$, conditions (2.4) and (2.12) are similar in effect. Therefore it may be concluded that where a core is too weak to provide a significant contribution to the flexural rigidity of the sandwich, the shear stress may be assumed constant over the depth of the core. For a weak core, it is therefore permissible to write $E_c = 0$ in equations (2.2) and (2.8); the constant shear stress in the core is then given by:

$$\tau = \frac{Q}{D}\frac{E_f td}{2} \qquad (2.13)$$

The way the shear stresses are distributed across the section is illustrated in fig. 2.4.b.

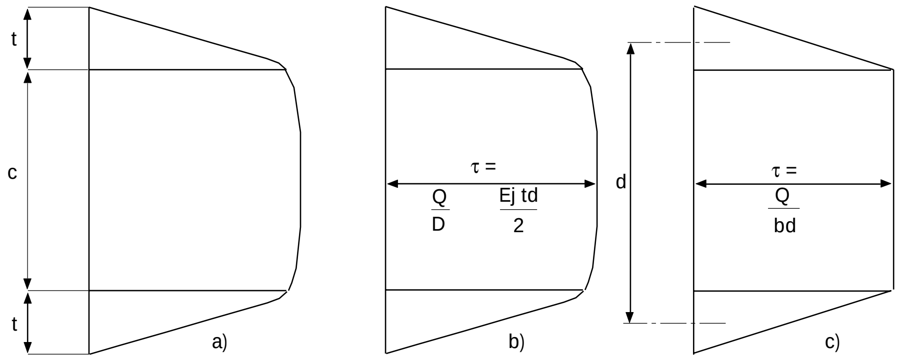


Fig. 2.4. Shear stress distribution in a sandwich beam.
(a) True shear stress distribution.
(b) Effect of weak core (conditions (2.4) and (2.12) satisfied).
(c) Effect of weak core, ignoring the local flexural rigidity of the faces (conditions (2.3), (2.4) and (2.12) satisfied).

If, in addition, the flexural rigidities of the faces about their own separate axes is small (i.e. if condition (2.3) is fulfilled), then the first term on the right-hand side of equation (2.2) may be ignored as well as the third, leaving:

$$D = E_f \frac{btd^2}{2} \qquad (2.14)$$

In this case equation (2.10) for the shear stress in the core is reduced to the simplest possible form:

$$\tau = \frac{Q}{bd} \qquad (2.15)$$

The corresponding shear stress distribution is illustrated in fig. 2.3c. The difference between fig. 2.3b and 2.3c is that in the latter the principle stress in each face is assumed to be uniform (because the local bending stress is ignored). It follows from this that the shear stress in the faces varies with depth in a linear fashion, not a parabolic one.

It is often convenient to invoke the concept of an "antiplane" core ($\sigma_x = \sigma_y = \tau_{xy} = 0$). An antiplane core is an idealised core in which the modulus of elasticity in planes parallel with the faces in zero but the shear modulus in planes perpendicular to the faces is finite. By this definition $E_c = 0$ and the antiplane core makes no contribution to the flexural rigidity of the beam. Conditions (2.4) and (2.9) are automatically satisfied and the shear distribution is similar to that shown in fig. 2.3b.

DEFLECTIONS

a/ Symmetrical loads

The loads considered here are symmetrical, i.e. the load is symmetrical with respect to the geometry of the beam and/or a relative horizontal displacement of the faces is prevented somewhere (for example at a clamped end).

In this case the flexural rigidity of the sandwich and the shear stress in the core are defined by equations (2.14) and (2.15). The shear stress distribution appears in fig. 2.4c.

In the first instance the transverse displacements (w_1) of the beam may be calculated by the theory of bending, using the relationship (2.1). For example, fig. 2.5b shows the bending deformation of a simply supported beam with a central point load W. The points a,b,c, ... lie on the centrelines of the faces and the cross sections aa, bb, cc, ... rotate but nevertheless remain perpendicular to the longitudinal axis of the deflected beam. It is obvious that the upper face is compressed as the points a, b, c, ... move closer together, while the lower face is loaded in tension.

The shear stress in the core at any section is τ = Q/bd (equation (2.15b)). This is associated with a shear strain γ = Q/Gbd which like τ, is constant through the depth of the core; G is the shear modulus of the core material. These shear strains lead to a new kind of deformation illustrated in fig. 2.5c.

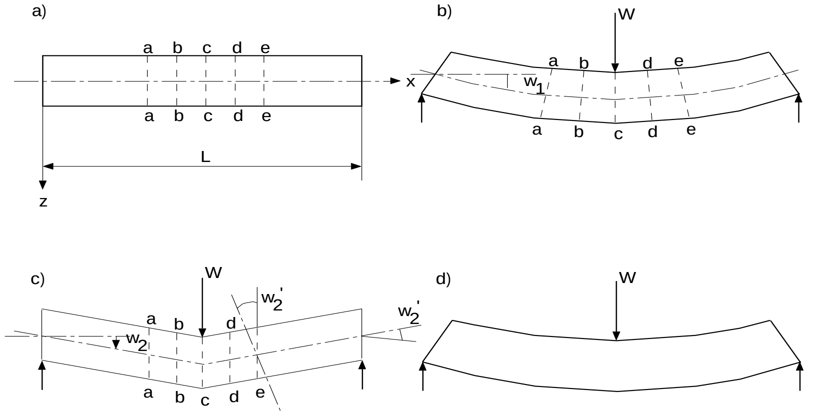


Fig. 2.5. Deflection of sandwich beam.

On the centrelines of the faces lie the points a, b, c, ... They are not moved horizontally but in a vertical direction w_2 due to shear strain. The faces and the longitudinal centreline of the beam tilt, and the relationship between the slope of the beam, dw_2/dx, and the core shear strain γ may be obtained from fig. 2.6. In this figure, which shows a deformation of a short length of the sandwich, the distance d e is equal to d(dw_2/dx). It is also equal to c f, which in turn is equal to γ c.

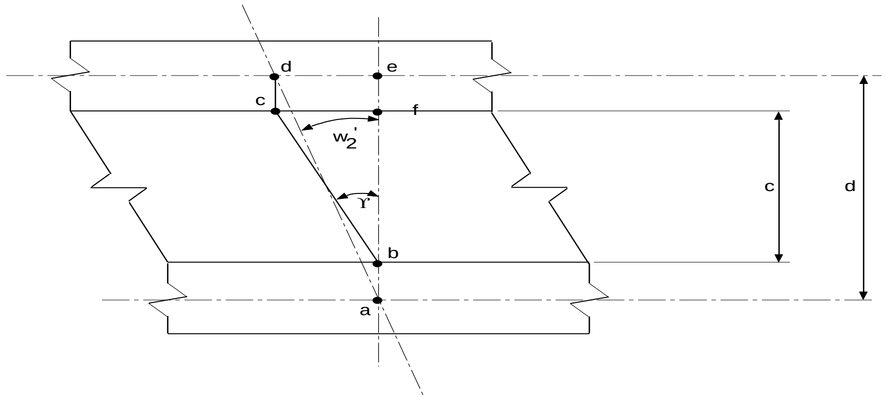


Fig. 2.6. Shear deformation of a beam.

Hence

$$\frac{dw_2}{dx} = \gamma\frac{c}{d} = \frac{Q}{Gbd}\,\frac{c}{d} = \frac{Q}{AG}\,\frac{c}{d} \qquad (2.16a)$$

Since the faces are assumed to be thin, c is about the same as d which means that $w_2' = \gamma$ and

$$w_2' = \frac{Q}{AG} = \frac{Q}{V} \qquad (2.16b)$$

The product V is often referred to as the shear stiffness of the sandwich. (The product also contains a factor called β but because of its rectangular shape, in this case it is 1.) The displacement w_2, associated with shear deformation on the core, may be obtained by integration of equation (2.16a) in any particular problem.

For example, in the simply supported beam with a central point load W, the transverse force Q in the left-hand half of the beam is + W/2. Integration of equation (2.16a) with Q = + W/2 provides the displacement:

$$w_2 = \frac{W}{2V}x + \text{constant} \qquad 0 \le x \le L/2$$

The constant vanishes because $w_2 = 0$ at $x = 0$. The maximum value of w_2 occurs at the centre of the beam, $x = L/2$, and is equal to:

$$\Delta_2 = \frac{WL}{4V}$$

The total central deflection Δ is therefore the ordinary bending displacement Δ_1 with the displacement Δ_2 superimposed:

$$\Delta = \Delta_1 + \Delta_2 = \frac{WL^3}{48D} + \frac{WL}{4V}$$

In general the displacement of any symetrically loaded sandwich beam with an antiplane core and thin faces may be found by similarly superimposing the bending and shear deflections w_1 and w_2. The bending deflections are found in the usual way and the shear deflections by integrating equation (2.16a).It may be convenient to integrate equation (2.16b) in general terms with the following result:

$$w_2 = \frac{M}{V} + \text{constant} \tag{2.17}$$

For a simply supported beam with the origin at one support the constant is always zero. Consequently the shear displacement diagram is the same as the bending moment diagram, with a factor 1/V applied to it.

For example, a simply supported beam of span L with a uniformly distributed load q has a central bending deflection Δ_1 equal to + $5qL^4/384$ D. The bending moment at the centre is + $qL^2/8$ and the central shear deflection Δ_2 is therefore + $qL^2/8V$. The total deflection Δ at the centre is given by:

$$\Delta = \Delta_1 + \Delta_2 = \frac{5qL^4}{384\,D} + \frac{qL^2}{8V} \tag{2.18}$$

In the same way expressions for total deflections are obtained for other cases. At the end of this chapter a few of the most usual load cases are presented. The maximum values of bending moment and shear forces are presented and may be used to give the stresses in the core and in the faces.

For other cases an elementary table of load cases can be used. Insert the appropriate value for Q in (2.16a) and use boundary conditions to integrate the whole expression.

b/ Unsymmetrical load

In the previous section it was assumed that during shear deformation all points on the centrelines of the faces moved only in the vertical direction, as in fig. 2.5c. In general, it is possible for one face as a whole to move horizontally with respect to the other.

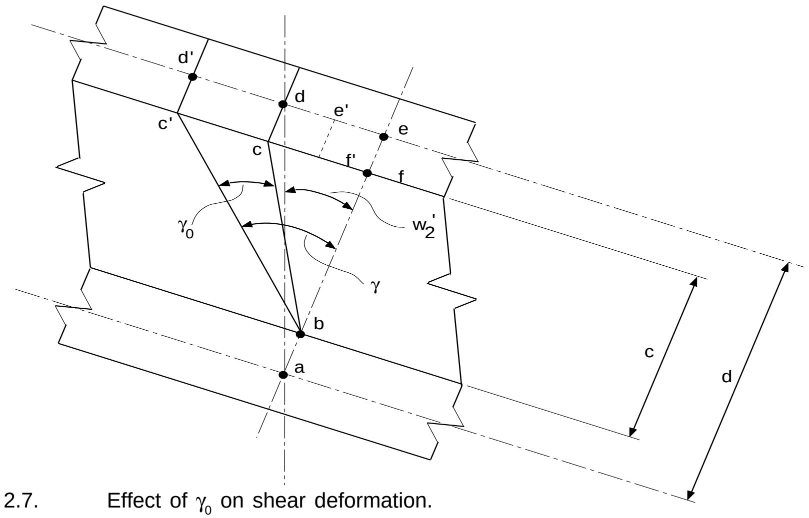


Fig. 2.7. Effect of γ_0 on shear deformation.

The effect is illustrated in fig. 2.7, which is similar to fig. 2.6 in showing the axis of the beam at an angle w_2' to the horizontal as a result of pure shear deformation of the core. However, the upper face has also been displaced to the left, so that the points cdef in figs. 2.6 and 2.7 now appear in new positions at c'd' e' f'. The angle cbc' is denoted by γ_0 and the following relationships exist:

$$cf = c'f - c'c = (\gamma - \gamma_0) \cdot c = de = w_2' \cdot d$$

Hence

$$w_2' = (\gamma - \gamma_0)\frac{c}{d} \tag{2.19a}$$

Or $\quad w_2{}' = \dfrac{Q}{AG} - \gamma_0 \dfrac{c}{d}$ (2.19b)

Or $\quad w_2 = \dfrac{M}{AG} - \gamma_0 \; x \; \dfrac{c}{d} + \text{constant}$ (2.19c)

Equations (2.16a) and (2.17) are merely special cases of (2.19b) and 2.19c).

Consider, for example, a simply-supported beam with a moment M_0 applied at one end (fig. 2.8). The bending moment at x is – M_0x/L, which value may be inserted in equation (2.19c):

$$w_2 = -\frac{M_0 x}{AGL} - \gamma_0 \; x \; \frac{c}{d} + \text{constant}$$

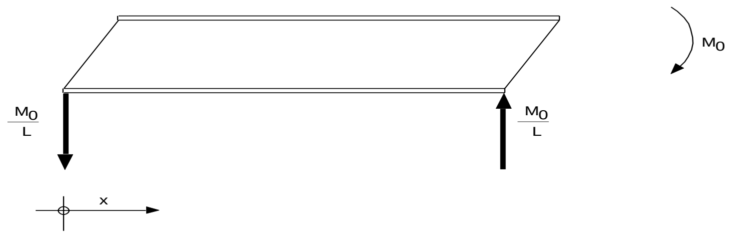


Fig. 2.8.

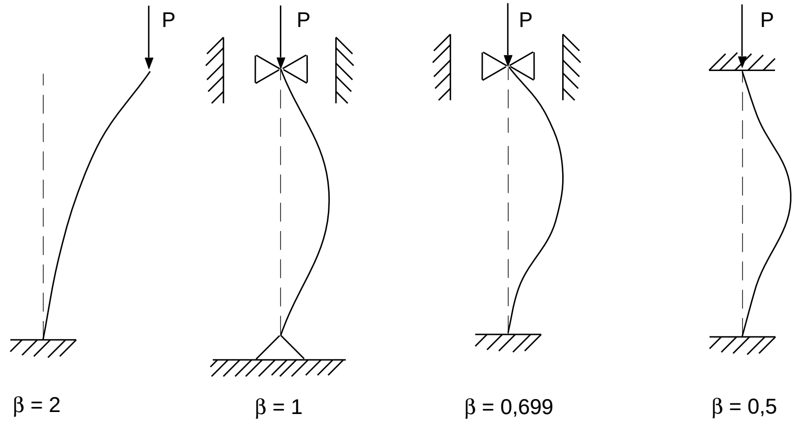
The boundary condition $w_2 = 0$ at x = 0, L shows that the constant vanishes and γ_0 is equal to – $M_0d/AGLc$. Substitution for γ_0 in equation (2.19c) shows that the transverse shear displacement w_2 is zero everywhere. However, all the sections through the core have rotated through an angle γ_0 as in fig. 2.8. The shear strain γ at all points in the core is given by equation (2.19a) as

$$\gamma = \frac{d}{c} w_2{}' + \gamma_0 = \gamma_0 = -\frac{M_0}{AGL} \frac{d}{c}$$

The rotation γ_0 is always zero when the beam is loaded in a symmetrical manner, or when the relative horizontal displacement of the faces is prevented, for example at a clamped end.

BUCKLING OF SANDWICH STRUTS

Standard analysis of uniform beams and struts has shown that instability appears when the axial load p reaches the value of the Euler load P_E. The Euler load is here presented in four different cases, the elementary Euler cases.



$$P_E = \frac{\pi^2 D}{(\beta L)^2}$$

Fig. 2.9. The Euler load for different cases

The Euler load represents the smallest value for an axial load P at which the strut will not return to straight condition after being displaced in lateral direction.

In the case of a sandwich strut the occurring shear deformations reduce the stiffness of the strut and the buckling load will be smaller than the corresponding Euler load.

A pin-ended sandwich strut will be considered here. The flexural rigidity is given by equation (2.5). When the axial thrust P reaches a critical value P_{cr}, the displacement consists of two superimposed displacements: w_1 (bending displacement) and w_2 (displacement associated with shear deformation of core). The buckled strut is shown in fig. 2.10. At a section x the bending moment M is, referring to equation (2.1):

$$M = P(w_1 + w_2) = -D_1 w_1'' \qquad (2.20)$$

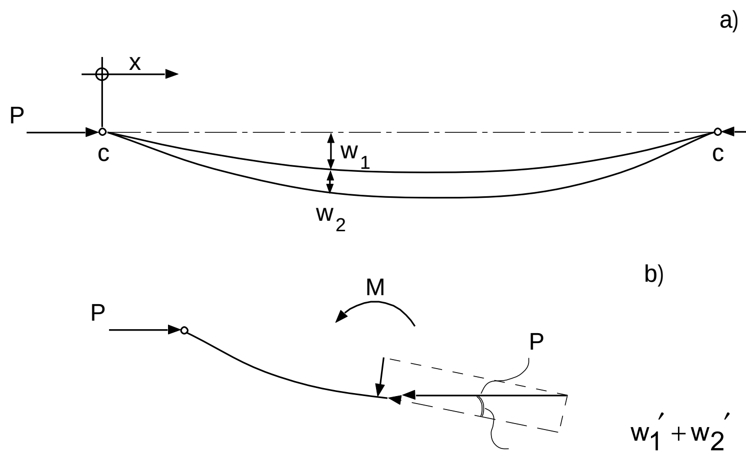


Fig. 2.10 Buckled strut with hinged ends.

Fig. 2.10 shows that P has a component $P(w_1' + w_2')$ acting perpendicular to the axis of the strut. This represents the transverse force. Corresponding to equation (2.16a) the shear force is related to w_2 by:

$$w_2' = \frac{P\left(w_1' + w_2'\right)}{V} \qquad (2.21)$$

The term w_2' may be eliminated from equations (2.20) (differentiated once) and (2.21) to yield a differential equation for w_1.

$$w_1'''' + \alpha^2 w_1'' = 0 \qquad (2.22a)$$

where

$$\alpha^2 = \frac{P}{D_1\,(1 - P/V)} \qquad (2.22b)$$

(2.22a) has a solution in the form:

$$w_1 = C_1 \sin\,\alpha x + C_2 \cos\alpha x + C_3 \qquad (2.23)$$

By differentiating (2.23) and inserting in the right-hand term of (2.21) the total deflection $w_1 + w_2$ will be obtained from

$$w_1 + w_2 = -\frac{D_1}{P}\left[-c_1\alpha^2 \sin\alpha x - C_2\alpha^2 \cos\alpha x\right] = +\frac{C_1 \sin\alpha x + C_2 \cos\alpha x}{1 + (P/V)} \qquad (2.24)$$

Boundary conditions provide that $C_2 = 0$ and if $(w_1 + w_2) = 0$ for $x = 0$ and $x = L$. This yields:

$$\alpha L = n(\mathrm{PI}) \qquad n = 1, 2, 3 \qquad (2.25)$$

Equation (2.22b) now yields:

$$P = \frac{P_E}{1 + P_E/V} \quad \text{where:} \quad P_E = \frac{\pi^2 D_1}{L^2} \qquad (2.26)$$

Where P represents the critical load P_{cr} of the sandwich strut. The expression is often given in this equal form:

$$\frac{1}{P_{cr}} = \frac{1}{P_E} + \frac{1}{V} \qquad (2.27)$$

In which is easily seen (V = AG):

* when G is finite, P_{cr} is less than the Euler load
* when G is infinite, P_{cr} is equal to the Euler load
* when G is small, P_{cr} approaches the value of AG.

These formulas can be used for all cases in fig. (2.9) with the appropriate Euler load inserted.

FLEXURAL RIGIDITY

To get a really strong sandwich beam a ”boxed” section can be chosen. With faces on all four sides the shear stiffness will be higher and the shear deflection will be smaller, though not negligible.

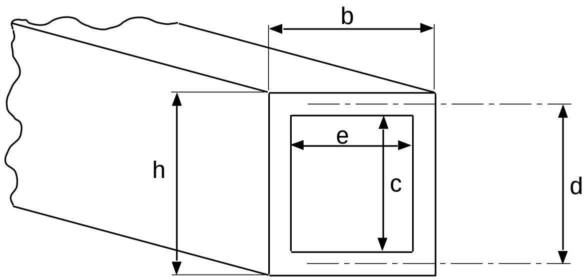


Fig. 2.11. Boxed beam

The expression for flexural rigidity in this case is:

$$D = E_f \frac{(bh^3 - ec^3)}{12} + E_c \frac{ec^3}{12} \qquad (2.28)$$

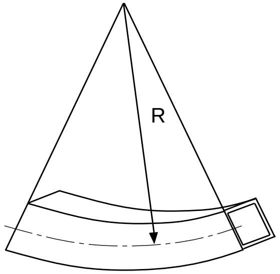
This is the flexural rigidity for bending about the centroidal axis of the cross section. Terms are the flexural rigidity of the box and the core respectively. If the second term amounts to less than 1% of the first it can be ignored. This means if:

$$\frac{E_f}{E_c}\left(\frac{bh^3}{ec^3} - 1\right) > 100 \qquad (2.29)$$

The second term is of no importance. Practically this is usually the case.

STRESSES IN BEAMS WITH FACES ON ALL FOUR SIDES

The stresses will in this case be calculated in the same way as before. However, first the bending moment distribution between faces and core must be found out. To evaluate an expression for the distribution of the bending moment a short beam section can be studied.



$$\kappa = \frac{1}{R}$$

Fig. 2.12.

The curvature is the same for the core and the face throughout the whole beam. Due to the theory of engineering stress in beams the curvature (κ) is given by the expression:

$$\kappa = \frac{M}{D} \qquad (2.30)$$

Hence

$$M_f = \kappa E_f I_f$$

$$M_c = \kappa E_c I_c$$

It is easy to see that M_c amounts to less than 1% of M_f when condition (2.29) is fullfilled. In practical cases the ratio will be even smaller, a fact that leads to the assumption that the bending moment is taken up in the face material only.

The normal stress in the core is then approximately zero and in the faces the stresses are calculated by:

$$\sigma_f = \frac{Mz}{D} \cdot E_f \qquad (2.31)$$

The maximum values for σ_f, i.e on the top and bottom, are obtained with maximum values for z.

The shear stresses depend on transverse forces. They are, in the same way as the bending moment, taken up in both the core and the faces. A study of a thin boxed beam section show that the ability for the core to take up shear forces can be ignored.

The shear deformation γ is the same for face and core, and is given by:

$$\gamma_i = \frac{Q_i}{V_i} \qquad (2.32)$$

where V contains a form factor β (see 2.35) , related to each part of the beam. For the core, β is assumed to be 1 and for the faces it is given by:

$$\beta = \frac{A_f}{A_{web}} \qquad (2.33)$$

where A_{web} is the cross-section area of the sides. This gives

$$Q_f = V_f \cdot \gamma$$

$$Q_c = V_c \cdot \gamma$$

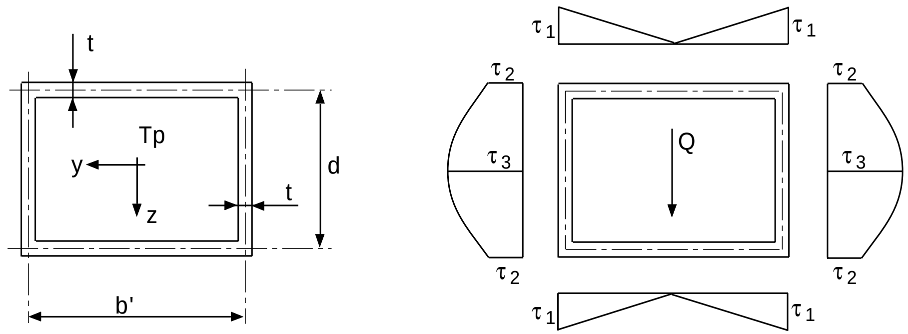
where the total shear force is:

$$Q = Q_f + Q_c$$

which means that Q_c amounts to less than 1% of Q_f if

$$\frac{V_c}{V_f} < \frac{1}{100} \qquad (2.34)$$

The condition will in most practical cases be fulfilled and Q_c can be ignored. Thus we have expressions for the different shear stresses in the faces. (See fig. 2.13)



$$\tau_1 = \frac{b'd}{4I_y} \cdot Q$$

$$\tau_2 = \frac{b'd}{4I_y} \cdot Q = \tau_1$$

$$\tau_3 = \frac{(2b' + d)d}{8I_y} \cdot Q$$

Fig 2.13 Shear stress distribution in a ”box”

DEFLECTIONS OF BEAMS WITH FACES ON ALL FOUR SIDES

Also in this kind of beam the deflection consists of two parts, bending deflection and shear deflection. In some case the shear deflection can be neglected, but the following example shows that this is not always the case.

Example:
A simple supported beam with faces on four sides is loaded by a concentrated load on the mid point of the beam. Face material is FRP (Fibre Reinforced Plastic) and the core is of Divinycell H 60. The deflection is given by expression (2.18). The first term is the bending deflection.

The flexural rigidity is given by (2.28) and with the length L = 1 m the bending deflection BD amounts to $3.1 \cdot 10^{-6} \cdot P$.

The deflection caused by shear deformation is given by the second term. Here the shear stiffness AG is denoted V. To calculate V we have to consider the shear stiffness of both the core and the side faces. When this is done the shear deflection SD amounts to $1.94 \cdot 10^{-7} \cdot P$.

The ratio SD/BD is 0.064, which means that the shear deflection is about 6% of the bending deflection and should not be ignored.

With a very long and slender beam the shear deflection can be ignored but in other cases the shear deflection must be considered. The shear stiffness was calculated:

$$V = V_c + V_f = \frac{G_c A_c}{\beta c} + \frac{G_f A_f}{\beta f} \qquad (2.35)$$

where β is a factor mentioned earlier by (2.33). For the core β_c is assumed to be 1 but for the faces β_f is given by (2.33). Due to equation (2.34) the first term in (2.35) can be ignored, leaving:

$$V = V_f = \frac{G_f A_f}{\beta f} \qquad (2.36)$$

BUCKLING BEAMS WITH FACES ON ALL FOUR SIDES

In the case of sandwich struts with faces on all four sides ("boxed struts") the calculations will be made in the same way as for ordinary sandwich struts. The formulas (2.26) and (2.27) can be used, but with D calculating according to (2.28) and the shear stiffness AG, here denoted V, according to (2.36). Since the shear stiffness is highly increased compared with an open strut, the critical load is higher, usually close to Euler load.

BEAMS CONSIDERED NARROW OR WIDE

A beam is considered narrow when the width b is less than the core dept c. Then the lateral expansions and contractions of the faces in the y-direction, associated with the membrane stress in the x-direction, may take place freely without causing large shear strains in the core in the yz-plane. The stresses in the faces are therefore mainly in one direction, and the ratio of stress to strain is equal to E. This has been assumed in the analysis of beams in this chapter.

The same argument does not apply to the local bending stresses in the faces. Each face is a thin plate in bending and the ratio of stress to strain is strictly $E/(1-\nu^2)$. However, these stresses and strains are of secondary importance and it seems reasonable to adopt E throughout in order to avoid complications.

A beam is considered wide when the width b >> the core depth c. Then lateral expansions and contractions of the faces in the y-direction are restricted by the inability of the core to undergo indefinitely large shear deformations in the yz-plane. In this case it is more reasonable to assume that the strains in the y-direction are zero. The ratio of stress to strain in the x-direction is therefore $E/(1-\nu^2)$ for both the membrane stresses and the local bending stresses. This value should be used in place of E in all equations of this chapter when a beam is considered wide. Note that if a wide beam can curve freely in the yz-plane, for instance if it is permitted to lift off its support, then E should be used in preference to $E/(1-\nu^2)$.

BEAMS WITH DISSIMILAR FACES

If the faces are not of the same material or of unequal thickness the results in chapters ”Flexural Rigidity” and ”Stresses” have to be modified. The principal beam equations are unchanged provided that the flexural rigidity is written as follows:

$$D = \frac{bd^2 E_1 E_2 t_1 t_2}{E_1 t_1 + E_2 t_2} + \frac{b}{12}\cdot\left(E_1 t_1^3 + E_2 t_2^3\right) \qquad (2.37)$$

where the suffixes 1 and 2 refer to the upper and lower faces respectively. It is here assumed that condition (2.4) is fulfilled and the contribution from the core to the flexural rigidity is negligible. If the local flexural rigidities for the faces are negligible, i.e. if the condition (2.3) is fulfilled for each of the faces, the second term in (2.37) can also be ignored.

Then (which is assumed in the analysis in chapter ”Analysis method for sandwich beams”) the flexural rigidity should be written as follows:

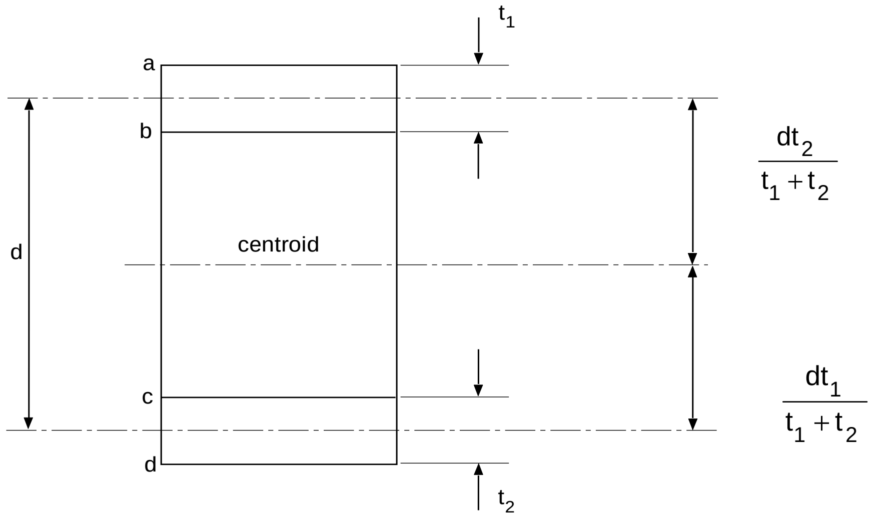


Fig. 2.14. Dimensions of sandwich with faces of unequal thickness.

$$D = \frac{bd^2 E_1 E_2 t_1 t_2}{E_1 t_1 + E_2 t_2} \qquad (2.38)$$

It is useful to note that equation (2.15) for the core shear stress is unaltered. d represents as usual the distance between the centroids of the upper and the lower faces.

BEAMS IN WHICH THE CONTRIBUTION TO THE FLEXURAL RIGIDITY FROM THE CORE IS NOT SMALL

When E_c is not small, i.e. when condition (2.12) is not fulfilled, some modification must be made to use chapter ”Open beams (free sides)”. For example the expression (2.2) must be used fully for flexural rigidity D. Since condition (2.12) is not satisfied, the shear stress τ and the shear strain γ are not to be considered constant throughout the depth of the core. The means equation (2.10) is valid but (2.13) is not.

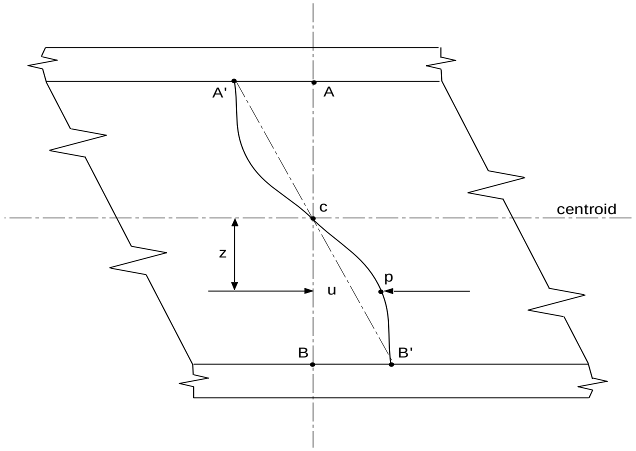


Fig. 2.15. Shear deformation of sandwich with stiff core

In fig. 2.15. a short length of a sandwich beam is shown undergoing shear deformation of the core.The section ACB has distorted into the curve A'CB'. The typical point p has moved a distance u to the right. At p the strain is γ = du/dz which gives the stress τ:

$$\tau = G\frac{du}{dz} \tag{2.39}$$

Equation (2.10) and (2.39) may be combined and integrated to yield and expression for u.

$$u = \frac{Q}{GD}\left[\frac{E_f tdz}{2} + \frac{E_c}{2}\left(\frac{c^2 z}{4} - \frac{z^3}{3}\right)\right] \tag{2.40}$$

For example the displacements AA' and BB' are obtained by writing z = ± c/2.

$$BB' = \frac{Q}{GD}\left(\frac{E_f}{4}tdc + \frac{E_c c^3}{24}\right) \tag{2.41}$$

The maximum shear stress is obtained by writing z = 0 in equation (2.10).

$$\tau_{max} = \frac{Q}{D}\left(\frac{E_f td}{2} + \frac{E_c c^2}{8}\right) \tag{2.42}$$

Now suppose that the core is replaced by a true antiplane ($\sigma_x = \sigma_y = \tau_{xy} = 0$) core with a shear modulus G', different from G^x, but keep the former D. The value of G' is chosen so that the section ACB is deformed to the straight line A'CB'. As the core is antiplane, E_c vanishes and the horizontal displacement becomes:

$$BB' = \frac{Q}{G'D}\left(\frac{E_f tdc}{4}\right) \tag{2.43}$$

Since G' has been chosen so that equations (2.39) and (2.41) give the same results for BB', the antiplane core (G') is exactly equivalent to the real core permitting us to use the analyses in chapter ”Analysis method for sandwich beams”. These analyses deal only with the core-edge displacements AA', BB' and do not depend on the shape of the distorted section A'CB'. Therefore the equivalent antiplane core has a shear modulus as follows:

$$G' = \frac{G}{1 + \frac{E_c}{6E_f}\cdot\frac{c^2}{t(c+t)}} \tag{2.44}$$

The procedure is now to use the analysis in chapter ”Analysis method for sandwich beams”, except that:

* D should be written as in equation (2.2)
* G is replaced with G'.

This procedure yields the correct deflections and stresses in the faces. To obtain the shear stress in the core equation (2.10) ((or (2.40)) should be used.

ANALYSIS METHOD FOR SANDWICH BEAMS

1 Check out in chapter ”Beams considered narrow or wide” if the beam is to be considered wide or narrow. In the following calculations the valid stress to strain ratio must be used.

2 If the faces are of unequal thickness the flexural rigidity is to be written as in equation (2.37).

3 Open beam
Check if condition (2.12) is fulfilled for the core to be considered weak, and if not,goto chapter ”Beams in which the contribution to the flexural rigidity from the core is not small” for proper adjustments.

Boxed beam
In most cases $G_f >> G_c$ and the shear stresses are taken up in the faces and are notconstant through-out the beam.

4 Flexural rigidity

Open beam:
use (2.2) and check condition (2.3) and (2.4) for ignoring terms.

Boxed beam:
use (2.28) and check condition (2.29) for ignoring the second term.

5 Shear stiffness
Open beam: use (2.16)
Boxed beam: use (2.35)

6 Stresses
a. Look in the elementary table for the moment and the transverse force.

b. Open beam:
σ_f from (2.7a)
σ_c from (2.7b)
τ_c from (2.10)
and check condition(2.12)
for ignoring the second term.

Boxed beam:
σ_f from (2.31)
τ_f from Fig. 2.14

7 Deflections
Open beams: see elementary table or use (2.16b) and (2.17)

Boxed beams: use elementary table or (2.16) and (2.17) with current rigidities according to point 4 and 5 above.

ASSUMPTIONS

The faces are assumed to be thin and of equal thickness.

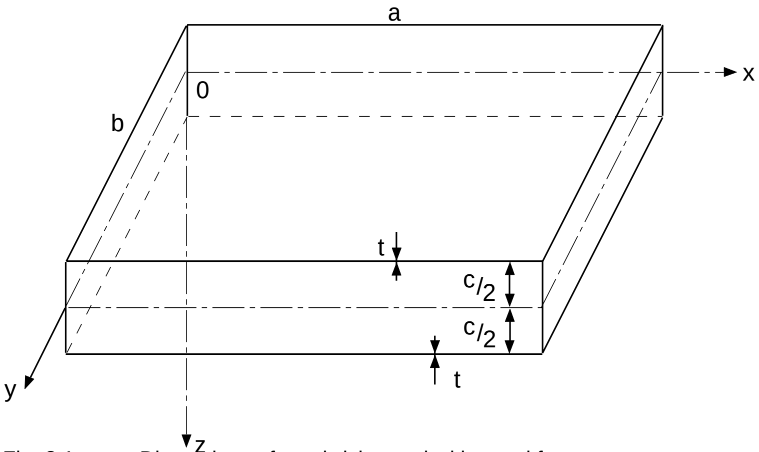


Fig. 3.1. Dimensions of sandwich panel with equal faces

Because the faces are thin compared to the core it is assumed that $c \approx d$ and that the local flexural rigidity of the faces is negligible. This means that the normal stress is constant throughout the faces. It is assumed that there are no stresses worth considering in the z-direction. The faces and the core are isotropic. The faces are assumed to be rigid in shear in yz- and zx-planes.

For the flexural rigidity of the panel, the core is assumed to be considerably less stiff than the faces. Consequently $E_c \sim 0$ in the xy-plane which leads to the fact that they do not contribute to the flexural rigidity. The core shear stresses are assumed to be constant throughout the depth of the core.

Further, the deflections are assumed to be small. Accordingly, ordinary bending theory is valid and there is no strain in the middle plane of the panel form transverse displacements, i.e.
$\sigma_y = \varepsilon_y = \sigma_x = \varepsilon_x = 0$ for $z = 0$.

SIGN CONVENTIONS

The sign convention that will be used for plates is shown in fig. 3.2:

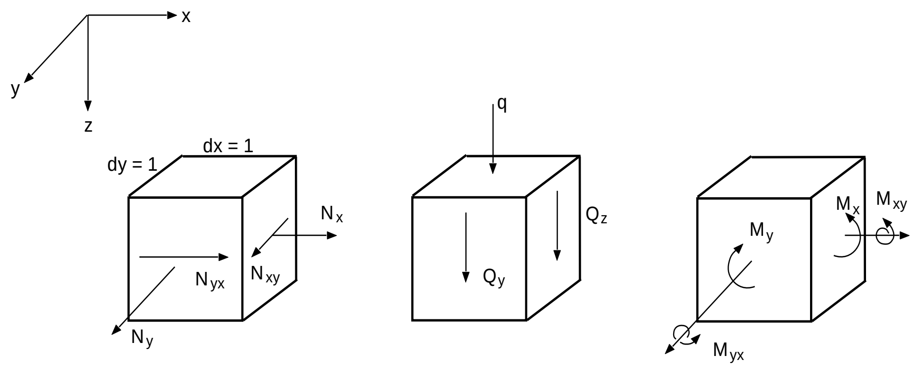


Fig. 3.2. Sign conventions for plates

The figure shows positive directions of bending and torsion moment (M_x, M_y, M_{xy}, M_{yx}), shear forces (Q_x, Q_y) and membrane forces (N_x, N_y, N_{xy}, N_{yx}).

BENDING AND BUCKLING OF SANDWICH PANELS SUPPORTED ON TWO SIDES

For sandwich panels supported on two opposite sides the theory and the formulas are the same as for open sandwich beams provided the load is a uniform pressure. However, it must be noted that the panel is considered as a wide beam due to chapter ”Beams considered narrow or wide”. Therefore, in the analysis E should be replaced by $E/(1-\nu^2)$. Assumptions are made similarly. The conditions used in chapter ”Open beams (free sides)” for ignoring terms when calculating flexural rigidity are the same.

From this it follows that in case of a sandwich panel supported on only two sides the reader is recommended to use chapter ”Open beams (free sides)” and the theory for open beams with E replaced by $E/(1-\nu^2)$.

BENDING AND BUCKLING OF PANELS SUPPORTED ON FOUR SIDES

For obtaining useful formulas, energy methods are applied to sandwich panels supported on all four edges.

The method aims to find expressions for the total potential energy in the material as a function of assumed displacements. The energy consists of two main parts: the strain energy U because of strain in core and faces of the deformed material, and the potential energy H because of movement of loads when deforming the panel.

The method is also based on the fact that the total energy (U + H) will have a minimum value when the deflected plate is in equilibrium. Accordingly the total energy (U + H) will be minimized with respect to deflection due to bending and shear to find the critical load, stresses and deflections. In fig. 3.3 a part of a deflected panel is shown.

The centre line AG and the normal AE have both rotated an angle ðw/ðx. Because of shear deformation the line AF has rotated a smaller angle λðw/ðx, where λ may take any value between + 1 and 0. From this is obtained the shear strain in the section (the angle EAF). λ = 1 means that the panel is rigid in shear and λ = 0 that there is no shear stiffness in the panel.

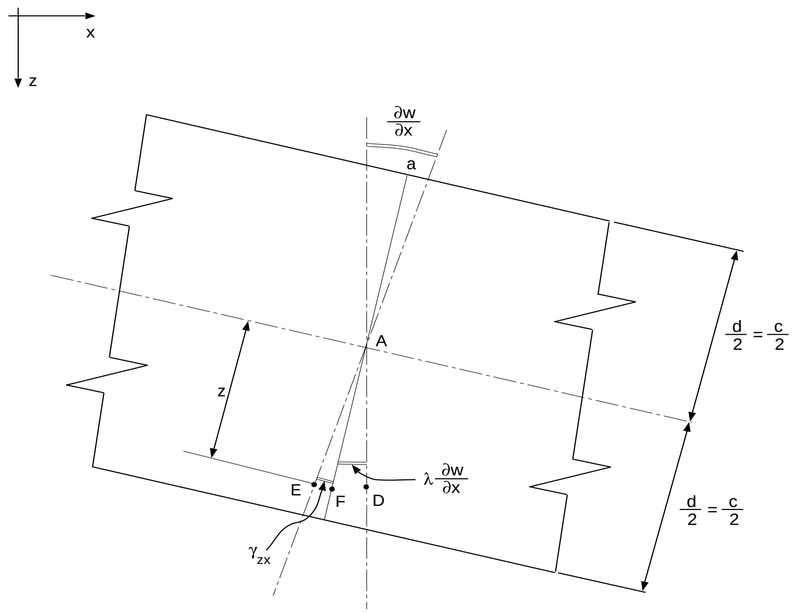


Fig. 3.3. Section through deflected sandwich panel in zx-plane

$$\gamma_{zx} = (1-\lambda)\frac{\partial w}{\partial x} \quad (3.1)$$

Since deformations are assumed to be small the displacement of F in the x-direction is:

$$u = -z\lambda\frac{\partial w}{\partial x} \quad 3.2)$$

In the same way

$$\gamma_{yz} = (1-\mu)\frac{\partial w}{\partial y} \tag{3.3}$$

$$v = -z\mu\frac{\partial w}{\partial y} \tag{3.4}$$

Where μ is the term corresponding to λ and v is the displacement in y-direction.

The strains in x- and y-direction are given by the displacements:

$$e_x = \frac{\partial u}{\partial x} = -z\lambda\frac{\partial^2 w}{\partial x^2} \tag{3.5}$$

$$e_y = \frac{\partial v}{\partial y} = -z\mu\frac{\partial^2 w}{\partial y^2} \tag{3.6}$$

The shear strain in the xy-plane is:

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -z(\lambda+\mu)\frac{\partial^2 w}{\partial x \partial y} \tag{3.7}$$

It must be added that λ and μ are treated as being independent of x and y during differentiation.

STRAIN ENERGY

The strain energy of an isotropic solid is given by integrating the strain energy over the volume:

$$U = \frac{E}{2g}\int_V (e_x^{\ 2} + e_y^{\ 2} + 2\nu\, e_x e_y)\, dV + \frac{G}{2}\int_V (\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2)\, dV \tag{3.8}$$

where $g = 1-\nu^2$.

This expression is to be used for core and faces respectively.

STRAIN ENERGY OF CORE, U_c

According to the assumptions, the strain energy from terms containing e_x, e_y and γ_{xy} will be zero. This leaves only shear strains γ_{zx} and γ_{yz} from (3.1) and (3.3) to be inserted in (3.8).

$$U_c = \frac{G_c}{2}\int_V \left[(1-\mu)^2\left(\frac{\partial w}{\partial y}\right)^2 + (1-\lambda)^2\left(\frac{\partial w}{\partial x}\right)^2\right] dV = \frac{G_c d}{2}\int_0^a\int_0^b \left[(1-\mu)^2\left(\frac{\partial w}{\partial y}\right)^2 + (1-\lambda)^2\left(\frac{\partial w}{\partial x}\right)^2\right] dy\, dx \tag{3.9}$$

where dV = dx dy dz.

STRAIN ENERGY OF FACES, U_f

According to assumptions γ_{yz} and γ_{zx} are zero. This leaves terms e_x, e_y and γ_{xy} to be inserted in (3.8) For the lower face z is + d/2 and the strain energy here is:

$$U_{lower} = \frac{E}{2g}\int_V \left[\frac{d^2}{4}\lambda^2\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{d^2}{4}\mu^2\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2\nu\frac{d^2}{4}\lambda\mu\left(\frac{\partial^2 w}{\partial x_2}\cdot\frac{\partial^2 w}{\partial y^2}\right)\right] dV + \frac{G_f}{2}\int_V \frac{d^2}{4}(\lambda+\mu)^2\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 dV \tag{3.10}$$

The total strain energy of both faces, U_f, is obtained by integrating over the thickness t and doubling. It is also convenient to write G = E/{2(1+ν)}.

$$U_f = \frac{Ed^2t}{4g}\int_0^a\int_0^b\left[\begin{array}{l}\lambda^2\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \mu^2\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\nu_f\lambda\mu\,\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} + \\ + \left(\frac{1-\nu_f}{2}\right)(\lambda+\mu)^2\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\end{array}\right]dy\,dx \qquad (3.11)$$

POTENTIAL ENERGY OF APPLIED LOADS

When a beam of length L is given a transverse deformation w, the ends of the beam approach each other by an amount δ.

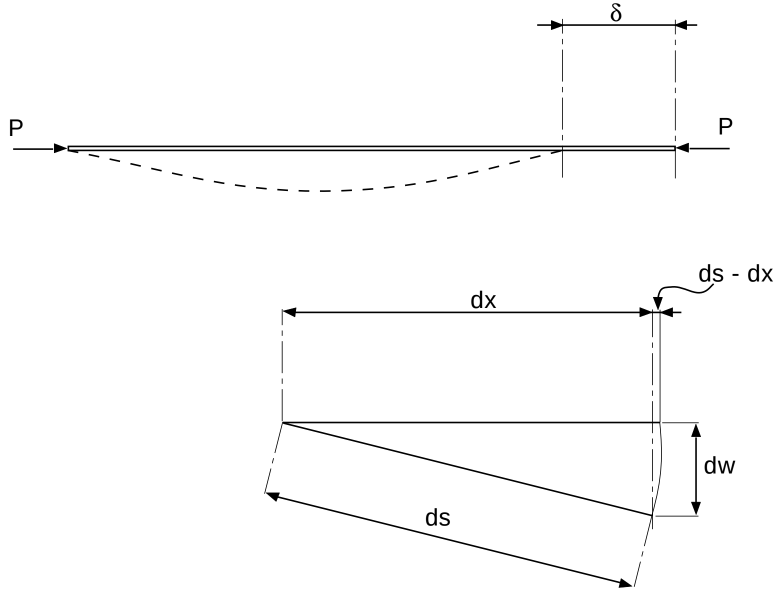


Fig. 3.4. Deformed beam and deformed element with length ds

ds in fig. 3.4 can be written:

$$ds = \sqrt{dx^2 + dw^2} = dx\sqrt{1+(dw/dx)^2}$$

and then developed in a series. Serial development yields:

$$dx\sqrt{1+(dw/dx)^2} = dx\left[1+\frac{1}{2}\left(\frac{dw}{dx}\right)^2 + \ldots\right]$$

For a beam element with the length ds the ends approach each other by an amount ds – dx. The equation above gives:

$$ds - dx = \frac{1}{2}\left(\frac{dw}{dx}\right)^2 dx$$

The total approach δ will be obtained by integrating over the length of the beam.

$$\partial = \frac{1}{2}\int_0^L\left(\frac{dw}{dx}\right)^2 dx$$

Consider now a narrow strip of the plate in fig. (3.1) parallel with the x-axis and of width dy. In the same way the ends of this strip approach each other as the plate bends by an amount:

$$\frac{1}{2}\int_0^a\left(\frac{\partial w}{\partial x}\right)^2 dx$$

If a compressive force N_x is applied at the edge (x = 0 and x = a) in the plane of the plate, then the force on the strip $N_x dy$ and the change in potential energy as the plate bends is:

$$-\frac{N_x dy}{2}\int_0^a\left(\frac{\partial w}{dx}\right)^2 dx$$

The total decrease in potential energy for the force N_x, is obtained by integrating from y = 0 to y = b.

$$V_1 = -\frac{N_x}{2}\int_0^a\int_0^b\left(\frac{\partial w}{\partial x}\right)^2 dx\ dy$$

If the plate also supports a uniform transverse pressure q in the z-direction, the decrease in potential energy V_2 for the load is:

$$V_2 = -\int_0^a\int_0^b wg\ dy\ dx$$

The displacement w for a simply-supported rectangular plate may be expressed by sums of trigonometric functions:

$$W = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{3.12}$$

Where a_{mn} is the amplitude of the (m, n)th mode of deformation. This expression satisfies the boundary conditions of a simply-supported plate.

The total energy of the system, (U + V), is obtained by adding the expressions for U and V respectively and substituting w by the series (3.12). Consider for example the first term of U_c.

$$U_c 1 = \frac{Gd^a}{2}\int_0^a\int_0^b\left[(1-\mu)\left(\frac{\partial w}{\partial x}\right)^2\right]dy\ dx \tag{3.13}$$

Substituting w according to (3.12) gives:

$$U_c 1 = \frac{Gd}{2}\int_0^a\int_0^b\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(1-\mu)_{mn}a_{mn}\frac{n\pi}{b}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\right]^2 dy\ dx \tag{3.14}$$

When the series is squared the integrals of the cross-product terms vanish because of the ortogonal properties of the chosen functions for w. Only the squared terms are left.

$$U_c 1 = \frac{Gd}{2}\int_0^a\int_0^b\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(1-\mu)^2 a_{mn}\left(\frac{n\pi}{b}^2\right)\sin^2\frac{m\pi x}{a}\cos^2\frac{n\pi y}{b}\right]dy\ dx \tag{3.15}$$

The series may be integrated term by term. The following integral is equal to ab/4 for all values of m and n.

$$\int_0^a\int_0^b \sin^2\frac{m\pi x}{a}\cos^2\frac{n\pi y}{b}dy\ dx = \frac{ab}{4} \tag{3.16}$$

Hence

$$U_{c1} = \frac{Gd}{2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(1-\mu_{mn})^2 a_{mn}^2\frac{n^2\pi^2}{b^2}\cdot\frac{ab}{4} \tag{3.17}$$

All the energy terms are to be treated in the same way. In this process it is useful to have knowledge about the following relationships, derived from equation (5.12).

$$\int_0^a\int_0^b\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dy\ dx = \sum\sum a_{mn}^2\frac{m^4\pi^4}{a^4}\cdot\frac{ab}{4}$$

$$\int_0^a\int_0^b\left(\frac{\partial^2 w}{\partial y^2}\right)^2 dy\ dx = \sum\sum a_{mn}\frac{n^4\pi^4}{a^4}\cdot\frac{ab}{4}$$

$$\int_0^a\int_0^b\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y_2}\right)dy\ dx = \sum\sum a_{mn}^2\frac{m^2n^2\pi^2}{a^2b^2}\cdot\frac{ab}{4}$$

$$\int_0^a\int_0^b\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 dy\,dx = \sum\sum a_{mn}^2\,\frac{m^{2n}2\pi^2}{a^2b^2}\cdot\frac{ab}{4}$$

$$\int_0^a\int_0^b\left(\frac{\partial w}{\partial y}\right)^2 dy\,dx = \sum\sum a_{mn}^2\,\frac{n^2\pi^2}{b^2}\cdot\frac{ab}{4}$$

$$\int_0^a\int_0^b w\,dy\,dx = \sum\sum \frac{4a_{mn}}{\pi^2}\cdot\frac{ab}{mn} \qquad \text{(m, n both odd)}$$

$$= 0 \qquad \text{(otherwise)}$$

By substituting these values in the former expressions for U_c, U_f, V_1 and V_2 the following expressions are obtained for the energy terms.

$$(U_c)_{mn} = GA_1\left((1-\lambda)^2\frac{m^2}{a^2}+(1-\mu)^2\frac{n^2}{b^2}\right)a_{mn}^2 \qquad (3.18a)$$

$$(U_f)_{mn} = EA_2\left(\lambda^2\frac{m^4}{a^4}+\mu^2\frac{n^4}{b^4}+2\nu\lambda\mu\frac{m^2n^2}{a^2b^2}+\frac{1-\nu}{2}(\lambda-\mu)^2\frac{m^2n^2}{a^2b^2}\right)a_{mn}^2 \qquad (3.18b)$$

$$(V_1)_{mn} = \frac{N_x}{2}\pi^2 a_{mn}^2\,\frac{ab}{4}\cdot\frac{m^2}{a^2} \qquad (3.18c)$$

$$(V_2) = -4q\frac{a_{mn}}{\pi^2}\cdot\frac{ab}{mn} \qquad (3.18d)$$

Where:

$$A_1 = \frac{d}{8}\pi^2 ab \quad \text{and} \quad A_2 = \frac{td^2}{16g}\pi^4 ab$$

For simplicity, only the (m, n)th mode is shown above and there are no suffixes on λ and μ. There are different values for each mode m, n.

Evidently (U + V) is a function of a_{mn}, λ and μ. If the plate is to be in equilibrium, (U + V) has to be stationary with respect to each of these variables. From this it follows that for each mode the following conditions must be fulfilled.

$$\frac{\partial}{\partial\lambda}(U+V) = \frac{\partial}{\partial\mu}(U+V) = \frac{\partial}{\partial a_{mn}}(U+V) = 0 \qquad (3.19)$$

These equations can be used to determine the values of a_{mn}, λ and μ. Since the (m, n)tn value of a_{mn}, λ and μ only appears in the (m, n)th mode, (U + V) in equation (3.19) could be replaced by $(U+V)_{mn}$ only.

It is easier to see the connections and follow the line of equations if the total energy is written in the form:

$$(U+V)_{mn} = B_{xx}\lambda^2 + B_{yy}\mu^2 + 2B_{xy}\lambda\mu + 2B_x\lambda + 2B_y\mu + B_0 \qquad (3.20)$$

Where:

$$B_{xx} = \left[GA_1\frac{m^2}{a^2}+EA_2\left(\frac{m^4}{a^4}+\frac{1-\nu}{2}\cdot\frac{m^2n^2}{a^2b^2}\right)\right]a_{mn}^2 \qquad (3.21a)$$

$$B_{yy} = \left[GA_1\frac{n^2}{b^2}+EA_2\left(\frac{n^4}{b^4}+\frac{1-\nu}{2}\cdot\frac{m^2n^2}{a^2b^2}\right)\right]a_{mn}^2 \qquad (3.21b)$$

$$B_{xy} = EA_2\left(\frac{1+\nu}{2}\right)\frac{m^2n^2}{a^2b^2}a_{mn}^2 \qquad (3.21c)$$

$$B_x = -GA_1\frac{m^2}{a^2}a_{mn}^2 \qquad By = -GA_1\frac{n^2}{b^2}a_{mn}^2 \qquad (3.21d, e)$$

$$B_0 = GA_1\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)a^2_{mn} + (V_1 + V_2)_{mn} \qquad (3.21f)$$

Equation (3.19) then gives:

$$\frac{1}{2}\frac{\partial}{\partial\lambda}(U+V) = \frac{1}{2}\frac{\partial(U+V)}{\partial\lambda} = B_{xx}\lambda + B_{xy}\mu + B_x = 0 \qquad (3.22a)$$

$$\frac{1}{2}\frac{\partial}{\partial\mu}(U+V) = \frac{1}{2}\frac{\partial(U+V)}{\partial\mu} = B_{yy}\mu + B_{xy}\lambda + B_y = 0 \qquad (3.22b)$$

If equations 3.22 are multiplied by λ and μ respectively and then added, then:

$$B_{xx}\lambda^2 + B_{yy}\mu^2 + 2B_{xy}\lambda\mu + B_x\lambda + B_y\mu = 0 \qquad (3.23)$$

(3.23) inserted in (3.20) leaves only:

$$(U+V)_{mn} = B_x\lambda + B_y\mu + B_0 \qquad (3.24)$$

By solving (3.22) and (3.22b) it is possible to show that in this particular problem the solution of equations (3.22) is such that $\mu = \lambda$.

Substitution for μ by λ in equation (3.22) gives the following result for λ:

$$\lambda = \frac{B_x}{Bx_x + Bx_y} = + \frac{1}{1 + \rho\Omega} \qquad (3.25a)$$

where

$$\Omega = \frac{m^2b^2}{a^2} + n^2 \qquad (3.25b)$$

$$\rho = \frac{\pi^2}{2g}\frac{E}{G}\frac{td}{b^2} \qquad (3.25c)$$

The variables λ, Ω and ρ are non-dimensional. λ and Ω take different values for different modes. ρ is constant and represents the ratio of the flexural rigidity $Etd^2/2g$ and the shear stiffness Gd.

When the expression for λ is inserted in (3.24) an expression for (U + V) as function of a_{mn} is given.

$$(U+V)_{mn} = \frac{Gd}{8}\pi^2 a^2_{mn}\frac{a}{b}\frac{\rho\Omega^2}{1+\rho\Omega} - \frac{N_x}{2}\pi^2 a^2_{mn}\frac{ab}{4}\frac{m^2}{a^2} - 4q\frac{a_{mn}}{\pi^2}\frac{ab}{mn} \qquad (3.26)$$

For equilibrium (3.26) must be stationary with respect to a_{mn}.

$$\frac{\partial}{\partial a_{mn}}(U+V)_{mn} = \left(\frac{Gd\pi^2}{4}\frac{a}{b}\frac{\rho\Omega^2}{1+\rho\Omega} - N_x\pi^2\frac{ab}{4}\frac{m^2}{a^2}\right)a_{mn} - \frac{4q}{\pi^2}\frac{ab}{mn} = 0 \qquad (3.27)$$

EDGE LOAD, N_x

Suppose that the transverse pressure q is zero. The critical load is then the value of N_x which causes the panel to buckle.

If the panel buckles in the (m, n)th mode a_{mn} is non-zero. Equation (3.27) is then satisfied only when:

$$N_x = \frac{Gd}{m^2}\left(\frac{a}{b}\right)^2\frac{\rho\Omega^2}{1+\rho\Omega} = P_{xmn} \qquad (3.28)$$

P_{xmn} is defined as the critical edge load per length unit which causes buckling in the (m, n)th mode. For any given m the lowest critical load is obtained for n = 1. Equation (3.28) can then be written as follows:

$$P_{xmn} = \frac{\pi^2 D_2}{b^2} K_1 \tag{3.29}$$

where:

$$K_1 = \frac{[(mb/a) + (a/mb)]^2}{1 + \rho\left[(mb/a)^2 + 1\right]} \tag{3.30}$$

And D_2 is the flexural rigidity of the sandwich:

$$D_2 = \frac{E_f t d^2}{2g} = \frac{E_f t d^2}{2\left(1 - \nu^2\right)} \tag{3.31}$$

Notice the factor g = (1–ν²) in the expression for the flexural rigidity of the plate. It originates from the conditions for a beam to be considered narrow or wide in ”Beams with odd properties” (see chapter ”Beams considered narrow or wide” and is always to be in the expressions in this chapter because panels are naturally considered wide.

It is observed that if shear rigidity is infinite, ρ vanishes and equation (5.30) is identical with the result for buckling of a plate not subjected to shear deformations.

Fig. 3.5 shows the value of K_1 plotted against a/b for m = 1,...,4 and four different values of ρ (0, 0.1, 0.2 and 0.4). Since only the lowest value of K_1 is of interest, only the lower envelopes of K_1 for m = 1,...,4 are to be used. Since the figure only shows four curves (m = 1,...4) it should be noticed that when a/b >> 1 the lower envelope of the curves get close to a straight horizontal line. The diagrams are valid for 0 < a/b < ~ 3.5 and for higher values the lowest value in the diagram can be used.

The procedure is to read K_1 from fig. 3.5 and then insert the value in equation (3.29) to determine the buckling load. If a value of ρ is obtained that it does not fit with the diagrams in fig. 3.5, the value for K_1 has to be calculated with (3.30).

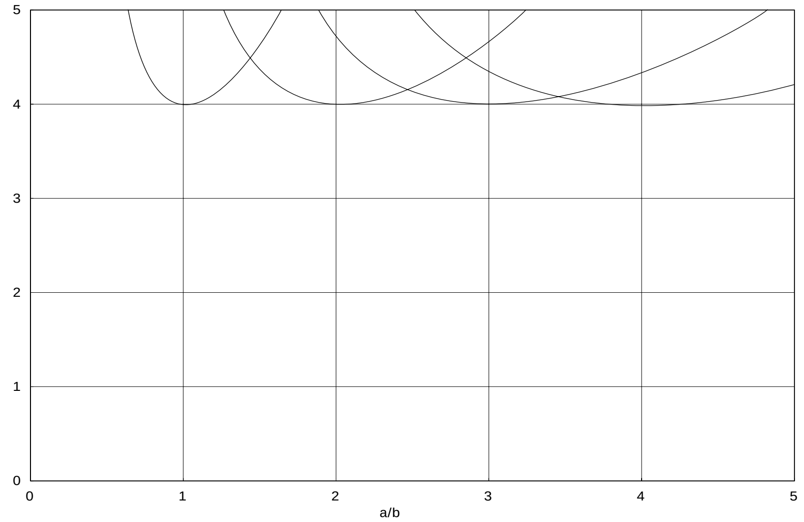


Fig. 3.5a. Buckling coefficient K_1 plotted against a/b for m = 1..4 and ρ = 0. Simply supported isotropic sandwich with thin faces.

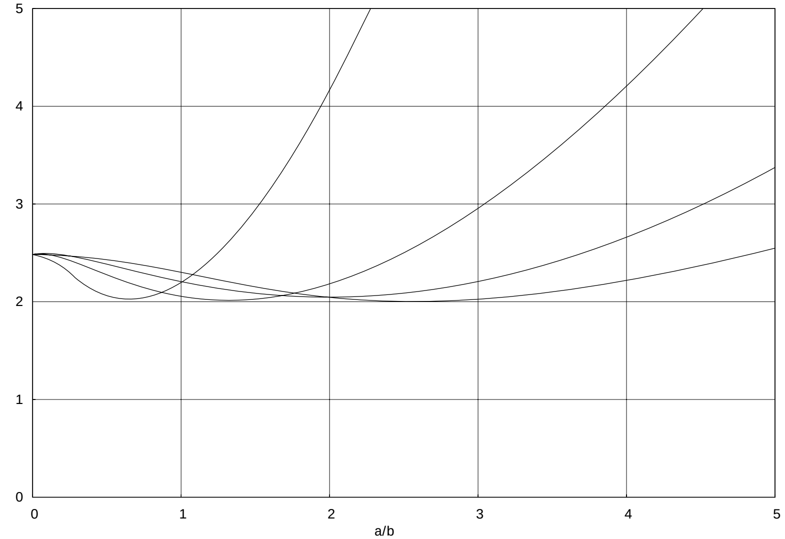


Fig. 3.5b. Buckling coefficient K_1 plotted against a/b for m = 1..4 and ρ = 0.1. Simply supported isotropic sandwich with thin faces.

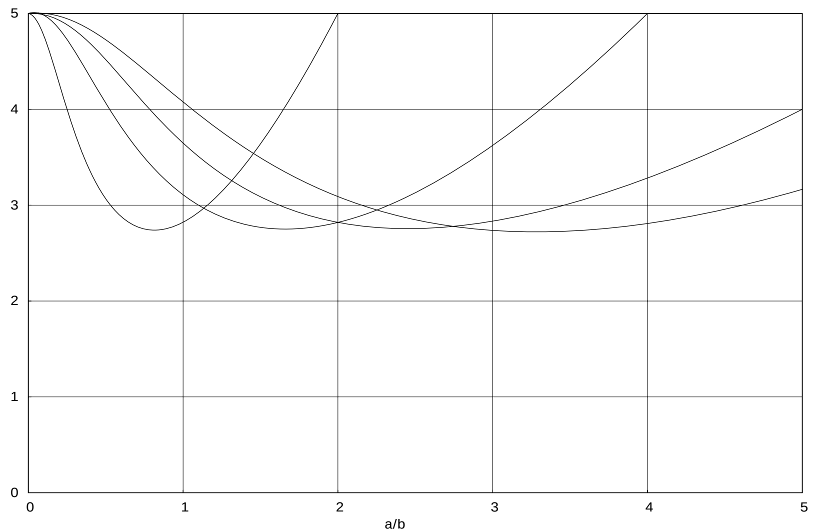


Fig. 3.5c. Buckling coefficient K_1 plotted against a/b for m = 1..4 and ρ = 0.2. Simply supported isotropic sandwich with thin faces.

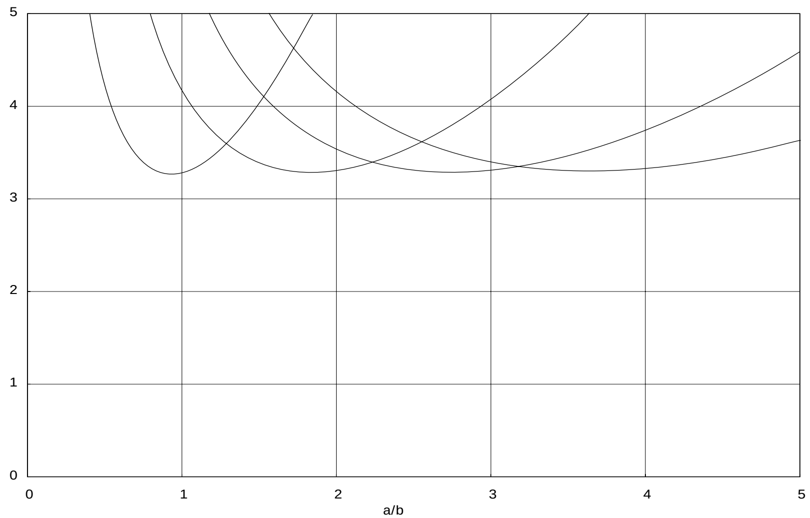


Fig. 3.5d. Buckling coefficient K_1 plotted against a/b for m = 1..4 and ρ = 0.4. Simply supported isotropic sandwich with thin faces.

UNIFORM PRESSURE, g

Now suppose that the edge load N_x is zero and that the load is q, a uniform pressure.

Substitution of ρ from equation (3.25c) provides an expression for the amplitude of the (m, n) th mode.

$$a_{mn} = \frac{16qb^4}{\pi^6 mnD_2} \cdot \frac{1+\rho\Omega}{\Omega^2} \quad \text{if m, n are both odd}$$

$$= 0 \quad \text{, otherwise.} \qquad (3.32)$$

In this case too, an infinite shear rigidity in the core causes ρ to vanish and then equation (3.32) corresponds to the standard result for bending of a plate not subjected to shear deformation.

Equation (3.32) may also be written in the form:

$$a_{mn} = \frac{16qb^4}{\pi^6 mnD_2\Omega^2} \cdot \frac{16qb^2}{\pi^4 mnGd\ \Omega} \quad \text{, if m, n are both odd} \qquad (3.33)$$

$$= 0 \quad \text{, otherwise}$$

The terms on the right side represent bending and shear deformations respectively. The ratio of the shear deformation to the bending deformation is

$$\rho\Omega \quad \text{, or} \quad \frac{1-\lambda}{\lambda} \quad \text{, or} \quad \frac{\pi^2 D_2}{b^2 Gd} \cdot \Omega$$

To obtain the deflection w the value a_{mn} must be inserted in equation (3.12). The maximum deflection w_{max} is at the centre of the panel, x = a/2, y = b/2, and is obtained by summation:

$$W_{max} = \frac{16qb^4}{\pi^6 D_2} \Sigma\Sigma \left[\frac{(-1)^{\frac{m-1}{2}} (-1)^{\frac{n-1}{2}}}{mn} \cdot \frac{1+\rho\Omega}{\Omega^2} \right] \qquad (3.34)$$

For practical use this expression may be written:

$$w_{max} = \frac{qb^4}{D_2}\left(\beta_1 + \rho\beta_2\right) \tag{3.35}$$

where:

$$\beta_1 = \frac{16}{\pi^6}\sum\sum\frac{(-1)^{\frac{m-1}{2}}(-1)^{\frac{n-1}{2}}}{mn\Omega^2} \quad \text{, m, n odd}$$

$$\beta_2 = \frac{16}{\pi^6}\sum\sum\frac{(-1)^{\frac{m-1}{2}}(-1)^{\frac{n-1}{2}}}{mn\Omega} \quad \text{, m, n odd}$$

β_1 and β_2 can be read from fig. 3.6 for panels with various a/b ratios.

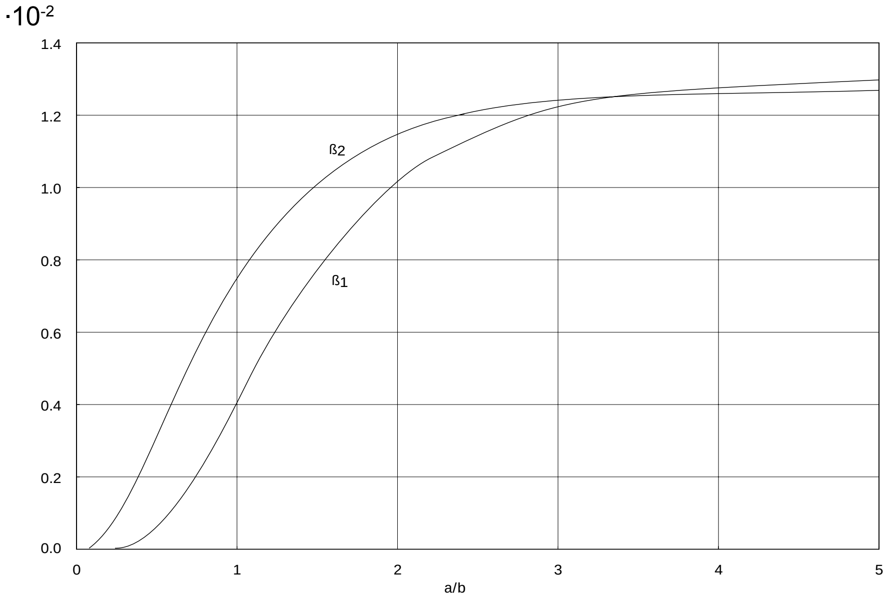


Fig. 3.6. Coefficients β_1 and β_2. Simply supported isotropic sandwich with thin faces.

The stresses in the faces and the core may also be obtained from (3.32). For example the normal stresses in the face (in x-direction) are equal to $(E/g)\cdot(e_x+\nu e_y)$. The strains e_x and e_y are defined in (3.5) and (3.6). By inserting z = ± d/2 the stresses in the x- and y-direction are

$$\sigma_x = \pm\frac{Ed\lambda}{2g}\left(\frac{\partial^2 w}{\partial x^2} + \nu_f\frac{\partial^2 w}{\partial y^2}\right) \tag{3.36a}$$

$$\sigma_y = \pm\frac{Ed\lambda}{2g}\left(\frac{\partial^2 w}{\partial y^2} + \nu_f\frac{\partial^2 w}{\partial x^2}\right) \tag{3.36b}$$

The shear stress τ_{xy} in the faces is equal to$[E/(2(1+\nu))]\, g_{xy}$ where the strain γ_{xy} is given by (3.7). For z = ± d/2 the shear stress is:

$$\tau_{xy} = \pm\frac{Ed\lambda}{2(1+v)}\cdot\frac{\partial^2 w}{\partial x\partial y} \tag{3.37}$$

The shear stress τ_{zx} in the core is equal to $G\gamma_{zx}$. When the strain γ_{zx} is given by (3.1).

$$\tau_{zx} = G(1-\lambda)\frac{\partial w}{\partial x} \tag{3.38a}$$

and similarly

$$\tau_{yz} = G(1-\lambda)\frac{\partial w}{\partial x} \tag{3.38b}$$

Usually the maximum stresses are of interest. For practical use it is convenient to write the expressions in the same way as equation (3.35).

It can be shown that the normal stresses in the faces are maximum at the centre of the panel (x = a/2, y = b/2). The shear stress in the faces is the highest at a corner (x = 0, y = 0), the core shear stress τ_{zx} is highest in the middle of the sides of length b (x = 0, y = b/2) and the core shear stress τ_{yz} is highest in the middle of the sides of length a (x = a/2, y = 0). The results may be summarized in the following forms:

$$\sigma_x = \frac{qb^2}{dt}\left(\beta_3 + \nu_f \beta_4\right) \tag{3.39a}$$

$$\sigma_y = \frac{qb^2}{dt}\left(\beta_4 + \nu_f \beta_3\right) \tag{3.39b}$$

$$\tau_{xy} = \frac{qb^2}{dt}\left(1 - \nu_f\right)\beta_5 \tag{3.39c}$$

$$\tau_{zx} = \frac{qb}{d}\beta_6 \tag{3.39d}$$

$$\tau_{yz} = \frac{qb}{d}\beta_7 \tag{3.39e}$$

where

$$\beta_3 = \Sigma\Sigma \frac{16}{\pi^4} \frac{(-1)^{\frac{m-1}{2}}(-1)^{\frac{n-1}{2}}}{\Omega^2} \cdot \frac{m}{n} \cdot \frac{b^2}{a^2} \tag{3.40a}$$

$$\beta_4 = \Sigma\Sigma \frac{16}{\pi^4} \frac{(-1)^{\frac{m-1}{2}}(-1)^{\frac{n-1}{2}}}{\Omega^2} \cdot \frac{n}{m} \tag{3.40b}$$

$$\beta_5 = \Sigma\Sigma \frac{16}{\pi^4} \cdot \frac{b}{a\Omega^2} \tag{3.40c}$$

$$\beta_6 = \Sigma\Sigma \frac{16}{\pi^3} \cdot \frac{(-1)^{\frac{n-1}{2}}}{n\Omega} \cdot \frac{b}{a} \tag{3.40d}$$

$$\beta_7 = \Sigma\Sigma \frac{16}{\pi^3} \cdot \frac{(-1)^{\frac{m-1}{2}}}{m\Omega} \tag{3.40e}$$

Fig. 3.7 shows $\beta_3 - \beta_7$ plotted against a/b. All the stresses are independent of shear stiffness of the core and it is possible to show that the results in equation (3.39) and (3.40) are the same as when the core shear deformation is ignored.

Results for a simply-supported rectangular panel can therefore also be used to calculate the stresses in a sandwich panel.

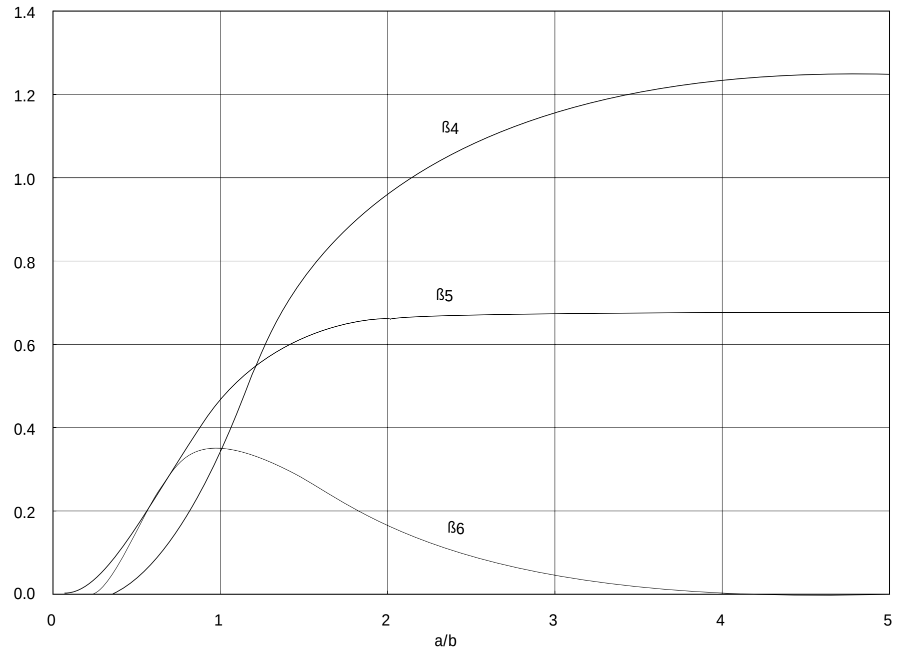


Fig. 3.7a. Constants $\beta_3 - \beta_5$. Simply supported isotropic sandwich with thin faces.

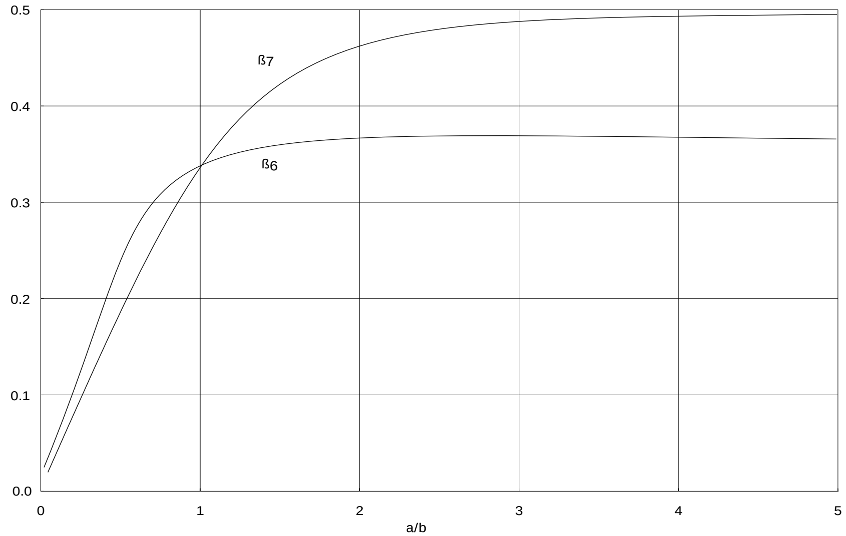


Fig. 3.7b. Constants β_6 and β_7. Simply supported isotropic sandwich with thin faces.

EDGE LOAD AND UNIFORM PRESSURE ACTING SIMULTANEOUSLY

When the uniform transverse pressure and the compressive edge load ρ per unit length act simultaneously, the value of a_{mn} can again be obtained from equation (3.27). The expression for a_{mn} is:

$$a_{mn} = \frac{(a_{mn})0}{1 - P/P_{xmn}} \qquad (3.41)$$

where $(a_{mn})_0$ is the amplitude when P is zero, given by equation (3.32) and P_{xmn} the critical load given by equation (3.29). Of course the expression is for the (m, n)th critical load.

The practical effect of this load arrangement is to multiply each term in the series for the β -functions by a factor $(1 - P/P_{xmn})^{-1}$. Because P_{xmn} depends on the ratio ρ, stresses in the panel are no longer independent of the shear stiffness.

PANELS WITH DISSIMILAR FACES

When the panels have faces of unequal thickness, or are of different materials, a few modifications have to be made. The buckling and bending equations (3.29) and (3.35) are unchanged provided both faces have the same Poisson's ratio ν_f and the following alterations are made:

$$D_2 = \frac{E_1 E_2 t_1 t_2 d^2}{\left(1-\nu_f^2\right)\cdot\left(E_1 t_1 + E_2 t_2\right)} \qquad (3.42)$$

$$\rho = \frac{\pi^2}{b^2\left(1-\nu_f^2\right)}\cdot\frac{E_1 E_2 t_1 t_2 c}{G\left(E_1 t_1 + E_2 t_2\right)} \qquad (3.43)$$

ANALYSIS METHOD FOR PANELS SIMPLY SUPPORTED ON FOUR SIDES

1: If the faces are of unequal thickness then modifications according to chapter ”Panels with dissimilar faces” have to be made.

2: Edge load, N_x. (buckling load)

The buckling load is given by (3.29) with flexural rigidity from (3.31), K_1 from fig. 3.5 as a function of a/b and ρ from (3.25c).

3: Uniform pressure, q. (Deflection and stresses)

Maximum deflection w_{max} is given by (3.35) with flexural rigidity from (3.31), the constants β_1 and β_2 from fig. 3.6 as a function of a/b and r from (3.25c).

The stresses are given by (3.39a-e) with the constants $\beta_3 - \beta_7$ from fig. 3.7 as a function of a/b.

4: Edge load and uniform pressure acting simultaneously.

Instructions written in chapter ”Edge loading and uniform pressure acting simultaneously”.

INTRODUCTION

In this section a few calculations are made to exemplify the use of the analysis methods presented in chapters ”Analysis method for sandwich beams” and "Analysis method for panels simply supported on four sides”. The expressions referred to are easy to find due to their numbering.

Example 1: Beam with concentrated load and simply supported ends

An open beam have the following measurements:

$L = 0.5$ m
$b = 0.05$ m
$h = 0.054$ m
$t_f = 2$ mm

The faces are of equal thickness and are made of aluminium 4054-7 with E_f = 61 GPa. The core is made of DIVINYCELL H 45 with the following properties:

$E_c = 45$ MPa
$G_c = 18$ MPa

The beam is simply supported at both ends and the load is a point load W = 25 kg at a = 0.25 · L = 0.125 m (see fig. 4.1). $L \geq a$.

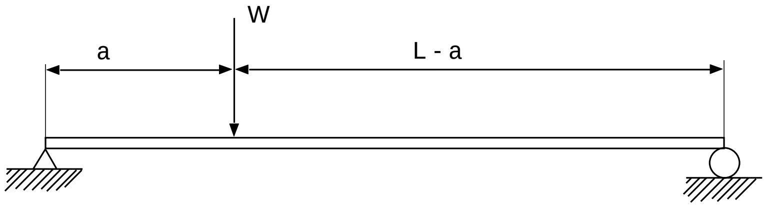


Fig. 4.1.

Find out σ_f, τ_c and the deflection at L/2 = 0.25 m. Solution according to chapter ”Analysis method for sandwich beams”.

1: $b \leq c \Rightarrow$ The beam is considered to be narrow.

2: The faces are of equal thickness.

3: Condition (2.12) is checked:

$$4\cdot\frac{61\cdot10^{9}}{40\cdot10^{6}}\cdot\frac{2\cdot10^{-3}}{50\cdot10^{-3}}\cdot\frac{52\cdot10^{-3}}{50\cdot10^{-3}} = 254 > 100$$

The condition is fulfilled E_c is considered small.

4: Flexural rigidity.
Conditions (2.3) and (2.4) are checked:

$$\frac{52\cdot10^{-3}}{2\cdot10^{-3}} = 26 > 5.77 \Rightarrow \text{condition (2.3) is fulfilled.}$$

$$\frac{61\cdot10^{9}}{40\cdot10^{6}}\cdot\frac{2\cdot10^{-3}\cdot\left(52\cdot10^{-3}\right)^2}{\left(50\cdot10^{-3}\right)^2} = 66.0 > 16.7 \Rightarrow \text{condition (2.4) if fulfilled.}$$

This leaves from equation (2.2) only the expression (2.6) fo the flexural rigidity D.

$$D = 61\cdot10^{9}\,\frac{50\cdot10^{-3}\cdot2\cdot10^{-3}\cdot\left(52\cdot10^{-3}\right)^2}{2} = 8247.2\,\text{Nm}^2$$

5: Shear stiffness.

$$(2.16) \Rightarrow V = bdG = 50\cdot10^{-3}\cdot52\cdot10^{-3}\cdot15\cdot10^{6} = 39\cdot10^{3}\,\text{N}$$

6: a/Elementary table gives:

$$M(L/2) = \frac{Wa\,(L-L/2))}{L} = \frac{245.25\cdot0.125\cdot(0.5-0.25)}{0.5} = 15.33\,\text{Nm}$$

$$Q\,(L/2) = (-)\frac{Wa}{L} = (-)\frac{245.25 \cdot 0.125}{0.5} = -\ 61.31\text{N}$$

b/Stresses
$\sigma_{f,max}$ is obtained with z = h/2

$$(2.7a)\ :\ \sigma_f = \frac{15.33 \cdot \left(54 \cdot 10^{-3}/2\right)}{8247.2} \cdot 61 \cdot 10^9 = 3.06\ \text{MPa}$$

τ_c is constant throughout the core as the condition (2.12) is fulfilled.

$$(2.13)\ :\tau_c = \frac{(-)61.31}{8247.2} \cdot \frac{61 \cdot 10^9 \cdot 2 \cdot 10^{-3} \cdot 52 \cdot 10^{-3}}{2} = 23.58\ \text{kPa}$$

7: Deflection.
Elementary table 1 gives:

$$w_1(L/2) = \frac{Wa}{48D} \cdot \left(3L^2 - 4a^2\right) =$$

$$= \frac{245.25 \cdot 0.125}{48 \cdot 8247.2} \cdot \left(3 \cdot 0.5^2 - 4 \cdot 0.125^2\right) = 0.053\ \text{mm}$$

$$w_2(L/2) = \frac{Wa}{2V} = \frac{245.25 \cdot 0.125}{2 \cdot 39 \cdot 10^3} = 0.393\ \text{mm}$$

Total deflection w

$$w = w_1 + w_2 = 0.45\ \text{mm}$$

Example 2: Beam with uniform pressure and clamped ends

The same beam as in example 1 is here loaded by a uniform load q = 1 kN/m and the ends are clamped.

Find out σ_f, τ_c and deflection at L/2 = 0.25 m.

Solution according to chapter "Analysis method for sandwich beams".

1: $b \le c \Rightarrow$ The beam is considered to be narrow.

2: The faces are of equal thickness.

3: Condition (2.12) is fulfilled (E_c is small). (see example 1)

4: Flexural rigidity.
D = 8247.2 Nm^2. (see example 1)

5: Shear stiffness.
V = $39 \cdot 10^3$ N (see example 1)

6: a/Elementary table gives: $M(L/2) = \frac{10^3 \cdot 0.5^2}{24} = 10.42\ \text{Nm}$

Q(L/2) = 0

b/Stresses.
$\sigma_{f,\,max}$ from (2.7a) with z = h/2.

$$\sigma_f = \frac{10.24 \cdot (0.054/2)}{8247.2} \cdot 61 \cdot 10^9 = 2.04\ \text{MPa}$$

(2.13) gives $\tau_c(L/2) = 0$ as Q(L/2) = 0.

At the ends $Q = \pm \frac{qL}{2} = \frac{10^3 \cdot 0.5}{2} = 250 \ \text{N}$, which with

(2.13) gives

$$\tau_c(L = 0) = \frac{250}{8247.2} \cdot \frac{61 \cdot 10^9 \cdot 0.002 \cdot 0.052}{2} = 96.15 \ \text{kPa}$$

Conclusion: it is important to study the shear force.

7: Deflection.
Elementary table gives:

$$w_1(L/2) = \frac{10^3 \cdot 0.5^2}{384 \cdot 8247.2} = 0.020 \ \text{mm}$$

$$w_2(L/2) = \frac{10^3 \cdot 0.5^2}{8 \cdot 39 \cdot 10^3} = 0.801 \ \text{mm}$$

Total deflection w.

$$w = w_1 + w_2 = 0.22 \ \text{mm}$$

Example 3: Boxed beam with concentrated load and simply supported ends

A boxed beam has the following measurements:

- $L = 0.5$ m
- $b = 0.05$ m
- $h = 0.054$ m
- $e = 0.05$ m
- $c = 0.046$ m
- $t_f = 2$ mm

The faces are of equal thickness and are made of FRP with the following properties:

- $E_f = 12$ GPa
- $G_f = 4.8$ GPa

Load and support are the same as in example 1.

Find out $\sigma_{f,\,max}$, $\tau_{c,\,max}$ and the deflection at $L/2 = 0.25$ m.

Solution according to chapter ”Flexural rigidity”.

1: $b \leq c \Rightarrow$ The beam is considered to be narrow.

2: The faces are of equal thickness.

3: $G_f >> G_c$.

4: Flexural rigidity.
Condition (2.29) is checked:

$$\frac{12 \cdot 10^9}{40 \cdot 10^6} \cdot \left(\frac{0.05 \cdot 0.054^3}{0.046 \cdot 0.05^3} - 1 \right) = 111 > 100$$

Which means that the expression (2.28) with the second term ignored can be used for the flexural rigidity.

$$D = 12 \cdot 10^9 \left(\frac{0.05 \cdot 0.054^3 - 0.046 \cdot 0.05^3}{12} \right) = 2123.2 \ \text{Nm}^2$$

5: Shear stiffness.

(2.33) gives: $\beta_c = \frac{4 \cdot 0.05 \cdot 0.002}{2 \cdot 0.05 \cdot 0.002} = 2$

then (2.35) gives:

$$V_c = \frac{G_c A_c}{\beta_c} = \frac{15 \cdot 10^6 \cdot 0.046 \cdot 0.050}{1} = 34.5 \cdot 10^3 \ \text{N}$$

$$V_f = \frac{G_f A_f}{\beta_f} = \frac{4.8 \cdot 10^9 \cdot 0.002 \cdot (2 \cdot 0.054 + 2 \cdot 0.046)}{2} = 960 \cdot 10^3 \ \text{N}$$

(2.34): $$\frac{V_c}{V_f} = \frac{34.5 \cdot 10^3}{960 \cdot 10^3} = 0.035 > \frac{1}{100}$$

The condition is not fulfilled, which means that the whole of expression (2.35) must be used.

$V = V_c + V_f = 994.5 \cdot 10^3$

6: Stresses

a/ $M(L/2) = 15.33$ Nm $Q(L/2) = -61.31$ N

b/ (2.31) gives with $z = h/2$ and $I = I_y = D/E_f = 0.1769 \cdot 10^{-6}$

$$\sigma_f = \frac{15.33 \cdot 0.054 / 2}{0.1769 \cdot 10^{-6}} = 2.34 \text{ MPa}$$

τ_{max} is given by fig. 2.13.

$$\tau_{max} = \tau_3 = \frac{(2b' + d)}{8 I_y} d \cdot Q =$$

$$= \frac{(2 \cdot 0.048 + 0.052)}{8 \cdot 0.1769 \cdot 10^{-6}} 0.052 \cdot -61.31 = 0.33 \text{ MPa}$$

7: Deflection

Elementary table 1 gives:

$$w_1(L/2) = \frac{Wa}{48D} \cdot \left(3L^2 - 4a^2\right) =$$

$$= \frac{245.25 \cdot 0.125}{48 \cdot 2123.2} \cdot \left(3 \cdot 0.5^2 - 4 \cdot 0.125^2\right) = 0.207 \text{ mm}$$

$$w_2(L/2) = \frac{Wa}{2V} = \frac{245.25 \cdot 0.125}{2 \cdot 994.5 \cdot 10^3} = 0.015 \text{ mm}$$

Total deflection w:

$$w = w_1 + w_2 = 0.22 \text{ mm}$$

Example 4: Panel with uniform pressure and simply supported edges

A panel has the following dimensions:

a = b = 3 m	h = 70 mm
t = 5 mm	c = 60 mm

The faces are of equal thickness and are made of FRP with $E_f = 12.0$ GPa and $\nu_f = 0.25$. The core has the following properties: $E_c = 200$ MPa, $G_c = 80$ MPa and $\nu_c = 0.30$. The panel is simply supported at all edges and the load is a uniform pressure q = 10 kPa.

Find out the maximum deflection and σ_f. Solution according to chapter "Analysis method for panels simply supported on four sides".

1: The faces are of equal thickness.

3: Flexural rigidity from (3.31).

$$D_2 = \frac{12.0 \cdot 10^9 \cdot 0.005 \cdot 0.065}{2 \cdot \left(1 - 0.25^2\right)} = 135.2 \cdot 10^3 \text{ Nm}^2$$

ρ from (3.25c) $$\rho = \frac{\pi^2 \cdot 12 \cdot 10^9 \cdot 0.005 \cdot 0.065}{2 \cdot \left(1 - 0.25^2\right) \cdot 80 \cdot 10^6 \cdot 3^2} = 0.0285$$

β_1 and b_2 are given in fig. 3.6 with a/b = 1

$\beta_1 = 0.42 \cdot 10^{-2}$ $\beta_2 = 0.73 \cdot 10^{-2}$

Then (3.35) gives

$$w_{max} = \frac{10 \cdot 10^3 \cdot 3^4}{135.2 \cdot 10^3} \cdot \left(0.405 \cdot 10^{-2} + 0.029 \cdot 0.74 \cdot 10^{-2}\right) = 25.6 \text{ mm}$$

σ_f from (3.39a) with β_3 and β_4 from fig. 3.7.a.

$\beta_3 = 0.0371$ $\beta_4 = 0.0385$ which gives

$$\sigma_x = \sigma_y \frac{10 \cdot 10^3 \cdot 3^2}{0.065 \cdot 0.005} \left(0.0371 + 0.25 \cdot 0.0385\right) = 12.94 \text{ MPa}$$

Example 5: Panel with edge load N_x and simply supported edges

The same panel as in example 4 but in this case the load is an edge load. Find out the buckling load.

Solution according to chapter ”Analysis method for panels simply supported on four sides”.

1: The faces are of equal thickness.

2: Flexural rigidity from (3.31)
$D_2 = 135.2 \cdot 10^3$ Nm^2 (see example 4)
ρ is given from (3.25c)
ρ = 0.0285 (see example 4)

Then K_1 is obtained from an interpolation between fig. 3.5a and 3.5b.
a/b = 1 $\Rightarrow$ K_1 = 3.78

The critical load is then given by (2.29):

$$P_{xmn} = \frac{\pi^2 \cdot 135.2 \cdot 10^3}{3^2} \cdot 3.78 = 560\,\text{kN/m}$$

Example 6: Beam with concentrated load, simply supported ends and stiff core

The same beam as in example 1 and with the same dimensions. The face material is the same but the core is made of Divinycell H 130.

L = 0.5 m	E_f = 61 GPa	E_c = 130 MPa
b = 0.05 m		G_c = 50 MPa
h = 0.054 m		
t_f = 2 mm		

The load is a concentrated load W = 25 kg (245.24N) at a = 0.25 L = 0.125 m.

Find out σ_f, σ_c, τ_c and deflection at L/2 = 0 .25 m.Solution according to chapter ”Flexural rigidity”.

1: b $\leq$ c $\Rightarrow$ The beam is considered to be narrow.

2: The faces are of equal thickness.

3: Condition (2.12) is checked.

$$4 \cdot \frac{61 \cdot 10^9}{130 \cdot 10^6} \cdot \frac{0.002}{0.05} \cdot \frac{0.052}{0.05} = 78.08 < 100$$

The condition is not fulfilled. Chapter ”Beams in which the contribution to the flexural rigidity from the core is not small” ,means that G has to be replaced with G’ according to (2.44).

$$G' = \frac{50 \cdot 10^6}{1 + \frac{130 \cdot 10^6}{6 \cdot 61 \cdot 10^9} \cdot \frac{0.05^2}{0.002 \cdot (0.002 + 0.05)}} = 49.6\text{ MPa}$$

4. Chapter ”Beams in which the contribution to the flexural rigidity from the core is not small” means that the whole expression (2.2) should be used when calculating the flexural rigidity.

$$D = 61 \cdot 10^9 \cdot \frac{0.05 \cdot 0.002^3}{6} + 61 \cdot 10^9 \cdot \frac{0.05 \cdot 0.002 \cdot 0.052^2}{2} +$$
$$+ 130 \cdot 10^6 \cdot \frac{0.05 \cdot 0.05^3}{12} = 8319.0 \text{ Nm}^2$$

5: Shear stiffness according to (2.16).
V = bgG’ = 0.05 · 0.052 · 49.6 · 10^6 = 129.0 · 10^3 N

6: Stresses

a/ Elementary table gives:
M(L/2) = 15.33 Nm (see example 1)
Q(L/2) = – 61.31 N (see example 1)

b/
(2.7a) gives with z = h/2:

$$\sigma_f = \frac{15.33 \cdot (0.054/2)}{8319.0} \cdot 61 \cdot 10^9 = 3.04 \text{ MPa}$$

(2.7b) gives with z = c/2:

$$\sigma_c = \frac{15.33 \cdot (0.05/2)}{8319.0} \cdot 130 \cdot 10^6 = 6.00 \text{ kPa}$$

(2.10) gives with z = 0:

$$\tau_c = \frac{-61.31}{8319.0} \cdot \left(61 \cdot 10^9 \cdot \frac{0.002 \cdot 0.052}{2} + \frac{130 \cdot 10^6}{2} \cdot \frac{0.05^2}{4} \right) = 23.68 \text{ kPa}$$

7: Deflection
Elementary table gives:

$$w_1 (L/2) = \frac{245.25 \cdot 0.125}{48 \cdot 8319.0} \cdot \left(3 \cdot 0.5^2 - 4 \cdot 0.125^2 \right) = 0.053 \text{ mm}$$

$$w_2 (L/2) = \frac{245.25 \cdot 0.125}{2 \cdot 129 \cdot 10^3} = 0.119 \text{ mm}$$

Total deflection: $w = w_1 + w_2 = 0.17$ mm

Example 7: Beam with concentrated load, simply supported ends and faces of unequal thicknesses

The same load case as in example 1, but now the faces are of unequal thicknesses. The thickness of the upper face is still 2 mm but the lower is 4 mm. The faces are made of aluminium 4054-7 and the core is made of Divinycell H 40. Measurements and properties:

L = 0.5 m	E_f = 61 GPa	E_c = 40 MPa
b = 0.05 m	t_{f1} = 2 mm	G_c = 15 MPa
h = 0.056 m	t_{f2} = 4 mm	
c = 0.05 m	d = 0.053 m	

Here the suffixes 1 and 2 represent the upper and the lower face respectively.

The beam is simply supported at both ends and the load is a concentrated load W = 25 kg (245.5 N) at a = L/4 = 0.125 m.

Find out t_c and deflection at L/2 = 0.25 m. Solution according to chapter ”Flexural Rigidity”.

1: $b \leq c \Rightarrow$ The beam is considered to be narrow.

2: The faces are of unequal thicknesses and therefore the flexural rigidity shall be written as in (2.37).

3: Condition (2.12) is checked (for the thinner face) in example 1and is fulfil led.

4: Flexural rigidity.
Condition (2.3) is checked for both of the faces.

$$\frac{0.053}{0.004} = 13.25 > 5.77 \qquad \frac{0.053}{0.002} = 26.50 > 5.77$$

The condition is fulfilled.Condition (2.4) is checked.

$$\frac{61 \cdot 10^9}{40 \cdot 10^6} \cdot \frac{0.002 \cdot 0.053^2}{0.05^3} = 68.54 > 16.7$$

which means that the condition is fulfilled for both of the faces.
Then the flexural rigidity is given by (2.38):

$$D = \frac{0.05 \cdot 0.053^2 \cdot \left(61 \cdot 10^9\right)^2 \cdot 0.002 \cdot 0.004}{61 \cdot 10^9 \cdot 0.002 + 61 \cdot 10^9 \cdot 0.004} = 11.42 \cdot 10^3 \text{ Nm}^2$$

5: Shear stiffness.
(2.16) gives:

$$V = bdG_c = 0.05 \cdot 0.053 \cdot 15 \cdot 10^6 = 39.75 \cdot 10^3 \text{ N}$$

6: a/Elementary table gives:
Q(L/2) = – 61.31 N (see example 1)

b/τ_c is according to chapter ”Beams with dissimilar faces” given by (2.15):

$$\tau_c = \frac{-61.31}{0.05 \cdot 0.053} = -23.14 \text{ kPa}$$

7: Deflection.
Elementary table gives:

$$w_1\left(L/2\right) = \frac{245.25 \cdot 0.125}{48 \cdot 11.42 \cdot 10^3} \cdot (3 \cdot 0.5^2 - 4 \cdot 0.125^2) = 0.038 \text{ mm}$$

$$w_2\left(L/2\right) = \frac{245.25 \cdot 0.125}{2 \cdot 39.75 \cdot 10^3} = 0.386 \text{ mm}$$

Total deflection w: $w = w_1 + w_2 = 0.42$ mm.

ANALYSIS OF SANDWICH BY FEM

In this chapter a metod to analyse sandwich stryctures by the Finite Element Method (FEM) is presented. The method is general and can be peformed using any of the commercially available FE-codes.

To obtain the right stiffness in shear in the FEM-model the material properties of the core have been modified according to the geometry of the sandwich. The method has been verified by comparing the results of a FEM-model of a panel to analytical calculated displacements, stresses and buckling loads. There are also verifying examples corresponding to the analytical examples in the chapter ”Examples”.

Sandwich panels have been treated in detail in chapter ”Sandwich panels”. There the following assumptions are made:

1. The stresses perpendicular to the plane of the panel are negligible both in the core and in the faces.

2. The material in both the core and the faces is isotropic.

3. In most cases the modulus of elasticity in the core is so low that the contribution to bending stiffness is negligible.

4. The displacements are small, meaning that the theory of bending is valid.

5. The faces are thin compared to the core. This means that the local flexural rigidity can be ignored and that $c \approx d$ (see fig. 5.1).

PRESENTATION OF METHOD

In principle a FEM-model of a sandwich structure can be built up in three different ways.

a/ Both faces and core are modelled of solid elements.
b/ The faces are modelled of shell elements and the core of solid elements. These are shown in fig. 5.1.
c/ Shell element with math model off shore.

If only solid elements are used the elements will have the same thickness as the faces and the core respectively. The elements modelling the faces will be very extended, as the faces are thin. However, the solid elements give a better result if they are cubic, i.e. all sides have about the same length. Consequently a very large number of elements is required if the sandwich has thin faces. Consequently the FE-model will have a great number of degrees of freedom and the calculation time will be unacceptable.

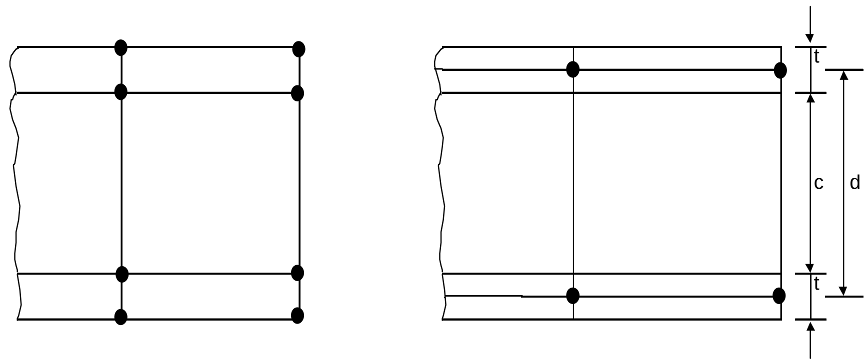


Fig. 5.1 Different ways of modelling sandwich by FEM. To the left with solid elements and to the right with a combination of solid elements and shell elements.

To avoid this problem and to reduce the number of degrees of freedom the model can be built up of shell elements representing the faces and of solids representing the core. As the nodes of a shell element are located in a plane in the middle of the element they should be placed at distance (c + d)/2 from each other. This is in accordance with assumption no 5 in chapter ”Analysis of sandwich by FEM”. The shear stiffness of the core will be reduced as the thickness of the core has increased by (d – c)/2 in the model. To compensate for this the shear modulus of the core can be increased to obtain the right stiffness in shear. The corrected shear modulus is given by the following expression:

$$G_{c,corr} = \frac{c + (d - c)/2}{c} \cdot G_c = \frac{d + c}{2c} \cdot G_c \qquad (5.1)$$

The stiffness in tension will be increased when the thickness of the core is increased; the modulus of elasticity should be corrected in the same way which gives:

$$E_{c,corr} = \frac{2c}{d + c} \cdot E_c \qquad (5.2)$$

In reality the modulus of elasticity has a very small influence on the deflections of a sandwich structure. However, it must be noted that if the modulus of elasticity and the shear modulus are corrected, the core material has to be modelled as orthotropic ((the expression $G = E/(2\,(1+\nu))$ is not valid)). Poisson’s ratios for faces and core respectively are assumed to be unaffected.

BEAM WITH CONCENTRATED LOAD AND SIMPLY SUPPORTED ENDS

$$Q(x)^{0-1} = \frac{Wb}{L} \qquad M(x)^{0-1} = \frac{Wbx}{L} \qquad M_{max} = M_1 = \frac{Wab}{L}$$

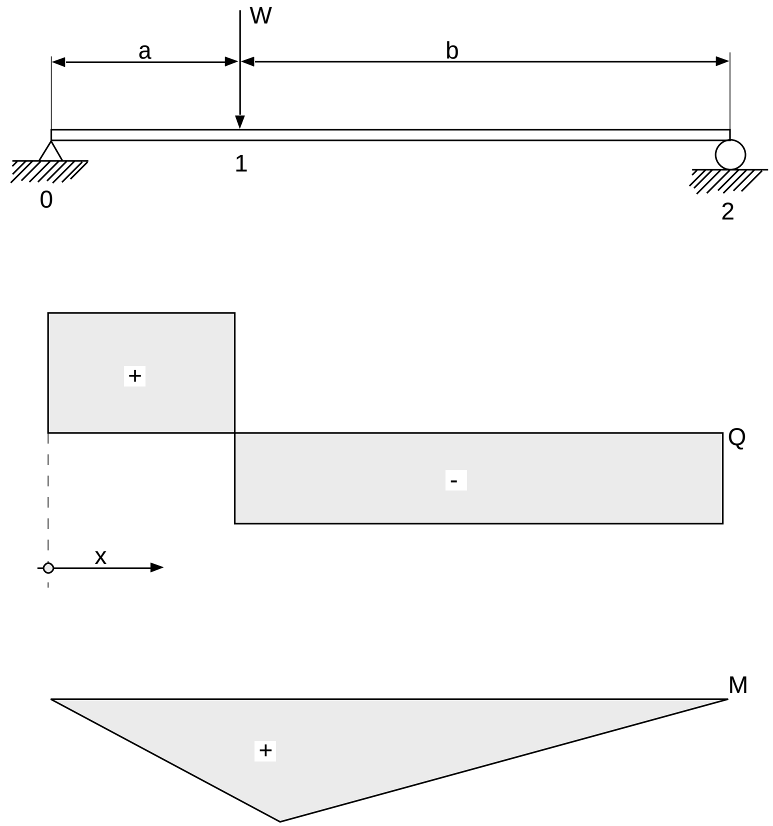
$$Q(x)^{1-2} = -\frac{Wa}{L} \qquad M(x)^{1-2} = \frac{Wa(L-x)}{L}$$

$$w_1(x)^{0-1} = \frac{WLbx}{6D}\left(1-\frac{b^2}{L^2}-\frac{x^2}{L^2}\right) \qquad w_2(x)^{0-1} = \frac{Wbx}{LV}$$

$$w_1(x)^{1-2} = \frac{WLa(L-x)}{6D}\left(\frac{2x}{L}-\frac{a^2}{L^2}-\frac{x^2}{L^2}\right) \qquad w_2(x)^{1-2} = \frac{Wa(L-x)}{LV}$$

$$w_1(a) = \frac{Wa^2b^2}{3DL} \qquad w_2(a) = \frac{Wba}{LV}$$

$$w_1(L/2) = \frac{Wa}{48D}\left(3L^2 - 4a^2\right) \qquad w_2(L/2) = \frac{Wa}{2V}$$



BEAM WITH UNIFORM PRESSURE AND SIMPLY SUPPORTED ENDS

$$Q(x) = q\left(\frac{L}{2} - x\right)$$

$$M(x) = \frac{qLx}{2} - \frac{qx^2}{2}$$

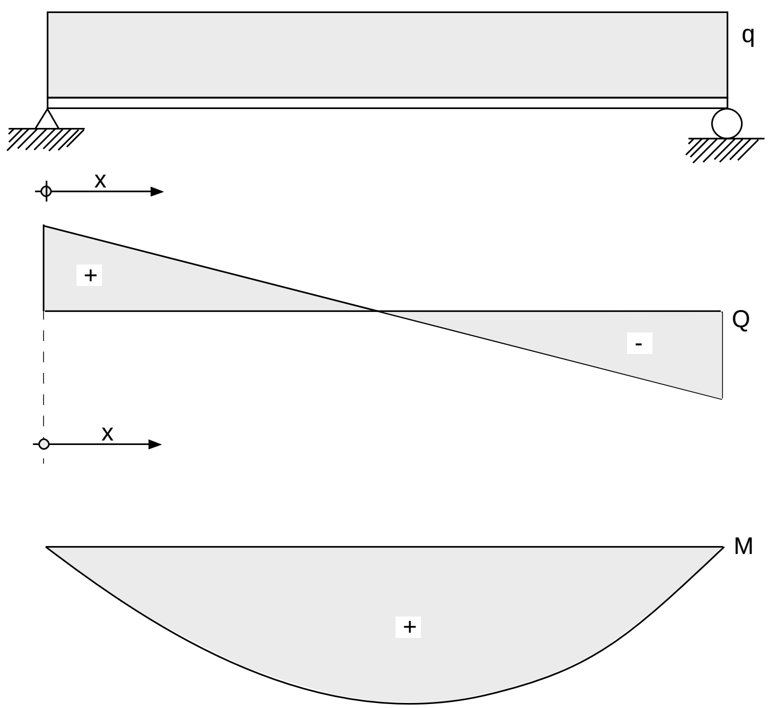
$$M(L/2) = M_{max} = \frac{qL^2}{8}$$

$$w_1(x) = \frac{qL^3x}{24D}\left(1 - 2\frac{x^2}{L^2} + \frac{x^3}{L^3}\right)$$

$$w_2(x) = \frac{q}{2V}\left(Lx - x^2\right)$$

$$w_{1,max} = \frac{5qL^4}{384D}$$

$$w_{2,max} = \frac{qL^2}{8V}$$



BEAM WITH TRIANGLE LOAD AND SIMPLY SUPPORTED ENDS

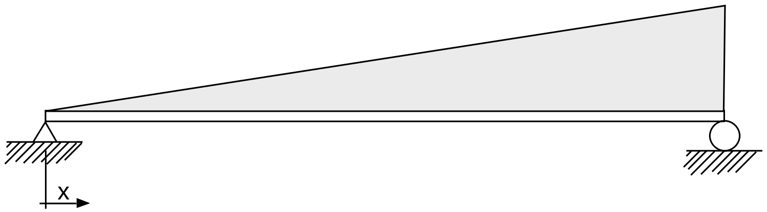
$$Q\,(x) = \frac{qL}{6} - \frac{qx^2}{2L} \qquad M\,(x) = \frac{qLx}{6}\left(1 - \frac{x^2}{L^2}\right)$$

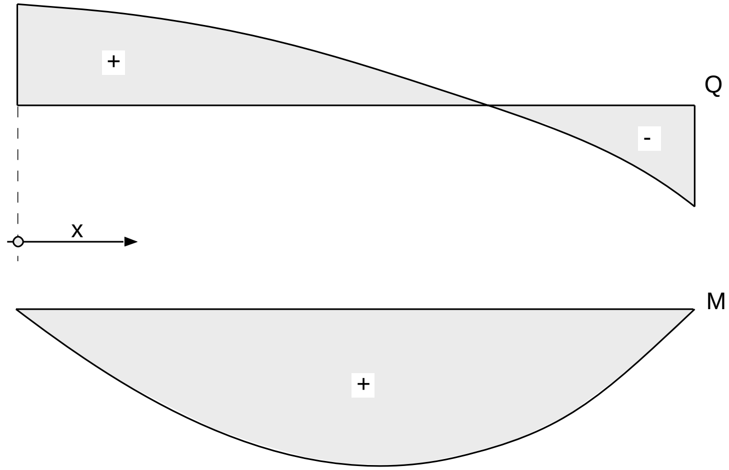
$$M\,(1/2) = \frac{qL^2}{16} \qquad M_{max} = 0.064\ qL^2 \text{ when } x = 0.577\ L$$

$$w_1\,(x) = \frac{qL^3x}{360D}\left(7 - 10\frac{x^2}{L^2} + 3\frac{x^4}{L^4}\right) \qquad w_2\,(x) = \frac{qLx}{6V}\left(1 - \frac{x^2}{L^2}\right)$$

$$w_1\,(1/2) = \frac{5qL^4}{768D} \qquad w_{2,max} \text{ appears at } x = 0.577\ L$$

$$w_{1,max} = 0.00652\,\frac{qL^4}{D} \text{ at } x = 0.519\ L$$





BEAM WITH CONCENTRATED LOAD, ONE SIMPLY SUPPORTED AND ONE CLAMPED END

$$Q\,(x)^{0-1} = \frac{Wb^2}{2L2}\left(3 - \frac{b}{L}\right)$$

$$M\,(a) = \frac{Wb^2 a}{2L^2}\left(2 + \frac{a}{L}\right)$$

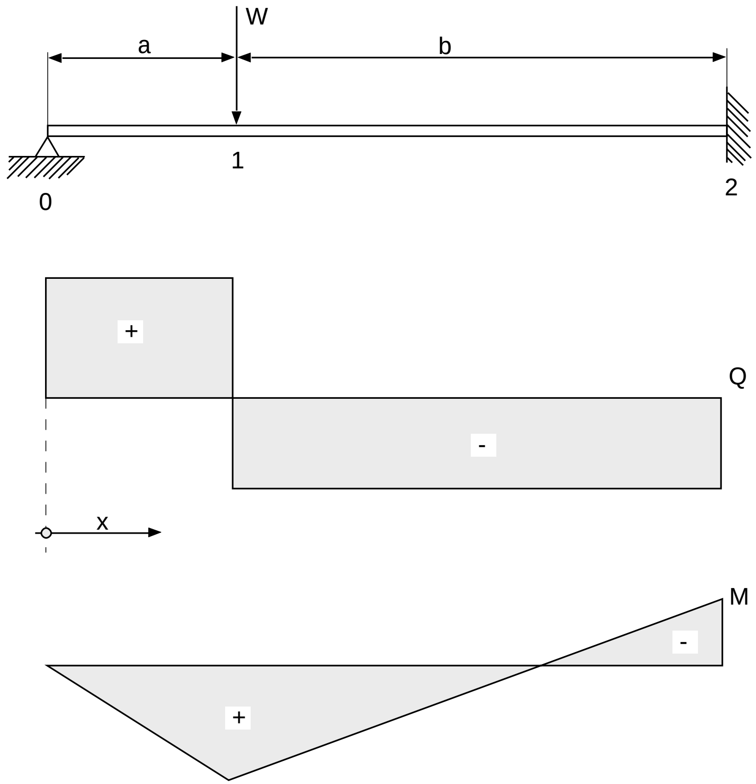
$$Q\,(x)^{1-2} = -\frac{Wa}{2L}\left(3 - \frac{a^2}{L^2}\right)$$

$$M\,(L) = -\frac{Wa}{2}\left(1 - \frac{a^2}{L^2}\right)$$

$$w_1\,(x)^{0-1} = \frac{Wb^2 x}{12D}\left[3\frac{a}{L} - \left(2 + \frac{a}{L}\right)\frac{x^2}{L^2}\right]$$

$$w_2\,(x)^{0-1} = \frac{Wb^2 x}{2L^2 V}\left(3 - \frac{b}{L}\right)$$

$$w_1\,(x)^{1-2} = \frac{Wa\,(L - x)^2}{12D}\left[3\left(1 - \frac{a^2}{L^2}\right) - \left(3 - \frac{a^2}{L^2}\right)\left(1 - \frac{x}{L}\right)\right]$$



$$w_{2,max} = w_2\,(a)$$

BEAM WITH UNIFORM PRESSURE, ONE SIMPLY SUPPORTED AND ONE CLAMPED END

$$Q(x) = \frac{3qL}{8} - qx$$

$$M(x) = \frac{gLx}{2}\left(\frac{3}{4} - \frac{x}{L}\right)$$

$$+M_{max} = \frac{9}{128} qL^2 \text{ at } x = 0.375\ L$$

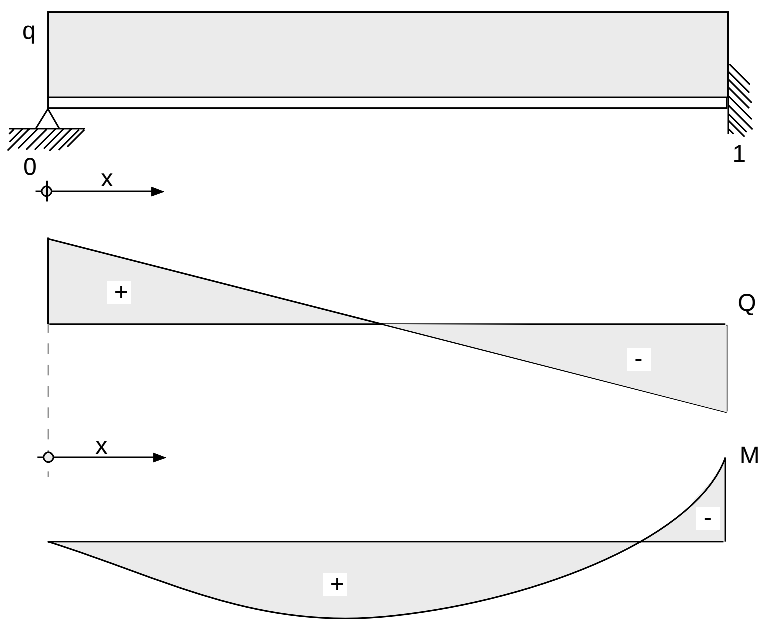
$$-M_{max} = M(L) = -\frac{qL^2}{8}$$

$$w_1(x) = \frac{qL^3 x}{48D}\left(1 - 3\frac{x^2}{L^2} + 2\frac{x^3}{L^3}\right)$$

$$w_2(x) = \frac{3qLx}{8V} - \frac{qx^2}{2V}$$

$$w_{1,max} = \frac{qL^4}{185D} \text{ at } x = 0.42\ L$$

$$w_1(L/2) = \frac{qL^4}{192D}$$



BEAM WITH TRIANGLE LOAD, ONE SIMPLY SUPPORTED AND ONE CLAMPED END

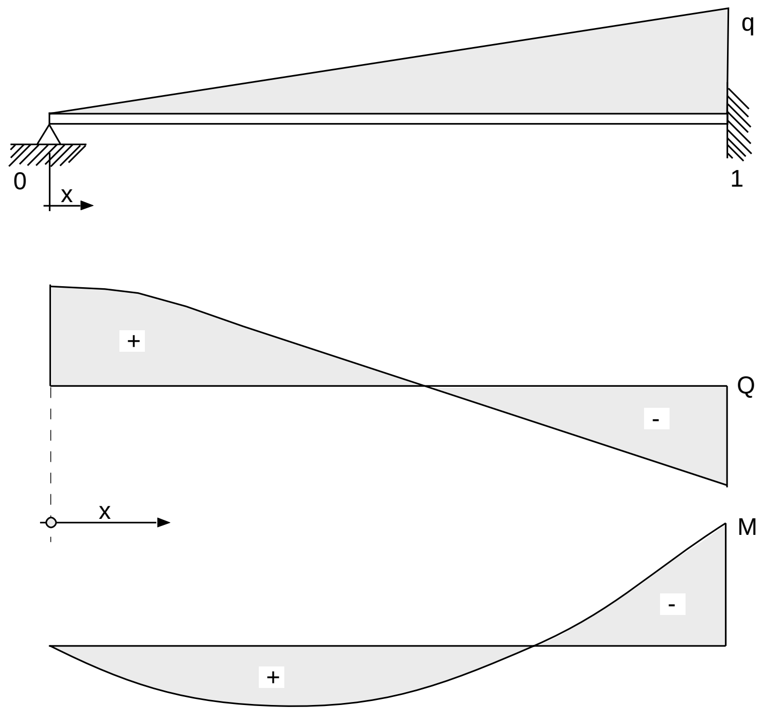
$$Q(x) = \frac{qL}{2}\left(\frac{1}{5} - \frac{x^2}{L^2}\right) \qquad M(x) = \frac{qLx}{2}\left(\frac{1}{5} - \frac{x^2}{3L^2}\right)$$

$$+M_{max} = 0.0298\ qL^2 \ \text{at}\ x = 0.447\ L$$

$$-M_{max} = M(L) = -\frac{qL^2}{15}$$

$$w_1(x) = \frac{qL^3x}{120D}\left(1 - \frac{2x^2}{L^2} + \frac{x^4}{L^4}\right) \qquad w_2(x) = \frac{qL}{2V}\left(\frac{x}{5} + \frac{x^3}{3L^2}\right)$$

$w_{1,max}$ at $x = 0.447\ L$



BEAM WITH CONCENTRATED LOAD AND CLAMPED ENDS

$$Q(x)^{0-1} = \frac{Wb^2}{L^2}\left(1 + \frac{2a}{L}\right) \qquad M(x)^{0-1} = -\frac{Wab^2}{L^2} + \frac{Wb^2x}{L^2}\left(1 + \frac{2a}{L}\right)$$

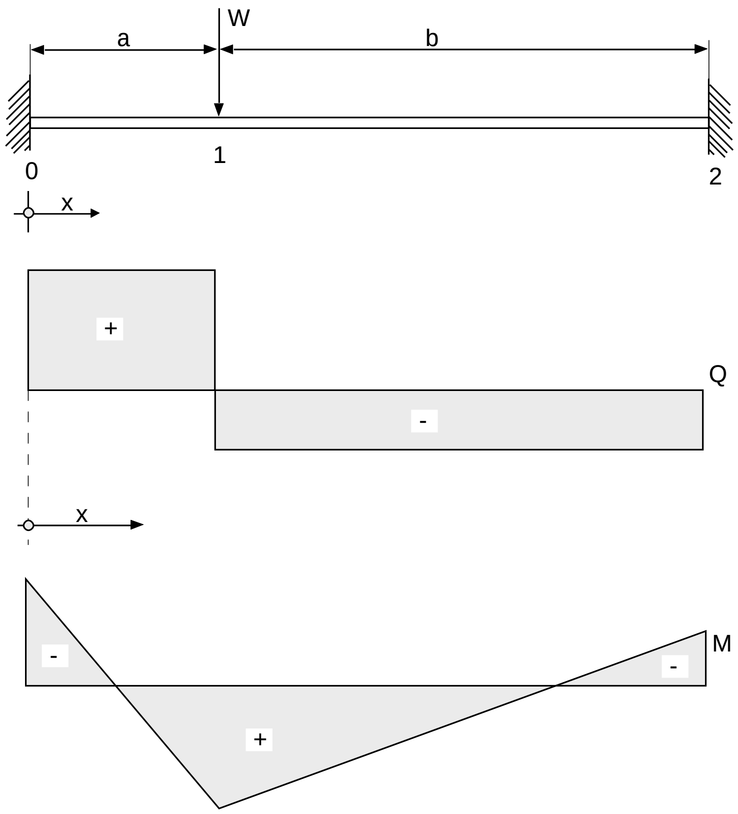
$$Q(x)^{1-2} = -\frac{Wa^2}{L^2}\left(1 + \frac{2b}{L}\right)$$

$$M(x)^{1-2} = -\frac{Wab^2}{L^2} + \frac{Wb^2x}{L^2}\left(1 + \frac{2a}{L}\right) - W(x-a)$$

$$w_1(x)^{0-1} = \frac{Wax^2}{6D}\left[3 - 6\frac{a}{L} + 3\frac{a^2}{L^2} - \frac{x}{a}\left(1 - 3\frac{a^2}{L^2} + 2\frac{a^3}{L^3}\right)\right]$$

$$w_1(L/2) = \frac{Wa^2L}{48D}\left(3 - \frac{4a}{L}\right)$$ when $a \leq L/2$

$$w_2(x)^{0-1} = \frac{Wb^2x}{VL2}\left(1 + \frac{2a}{L}\right)$$



BEAM WITH UNIFORM PRESSURE AND CLAMPED ENDS

$$Q(x) = \frac{qL}{2} - qx$$

$$M(x) = \frac{q}{2}\left(Lx - x^2 - \frac{L^2}{6}\right)$$

$$M(0) = M(L) = -\frac{qL^2}{12}$$

$$M(L/2) = \frac{qL^2}{24}$$

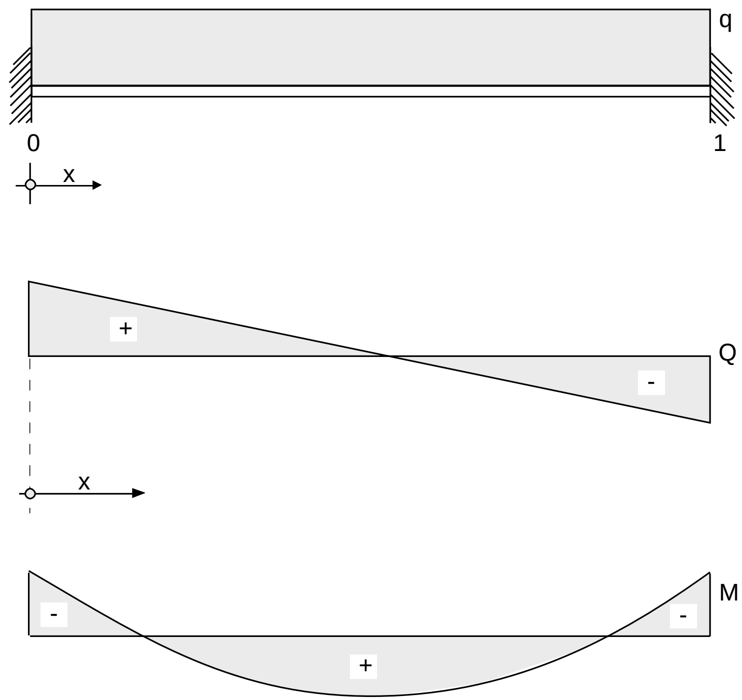
$$M = 0 \text{ at } x = 0.21\ L$$

$$w_1(x) = \frac{qL^2x^2}{24D}\left(1 - \frac{x}{L}\right)^2$$

$$w_2(x) = \frac{q(Lx - x^2)}{2V}$$

$$w_{1,max} = w1(L/2) = \frac{qL^4}{384D}$$

$$w_2(L/2) = \frac{qL^2}{8V}$$



BEAM WITH TRIANGLE LOAD AND CLAMPED ENDS

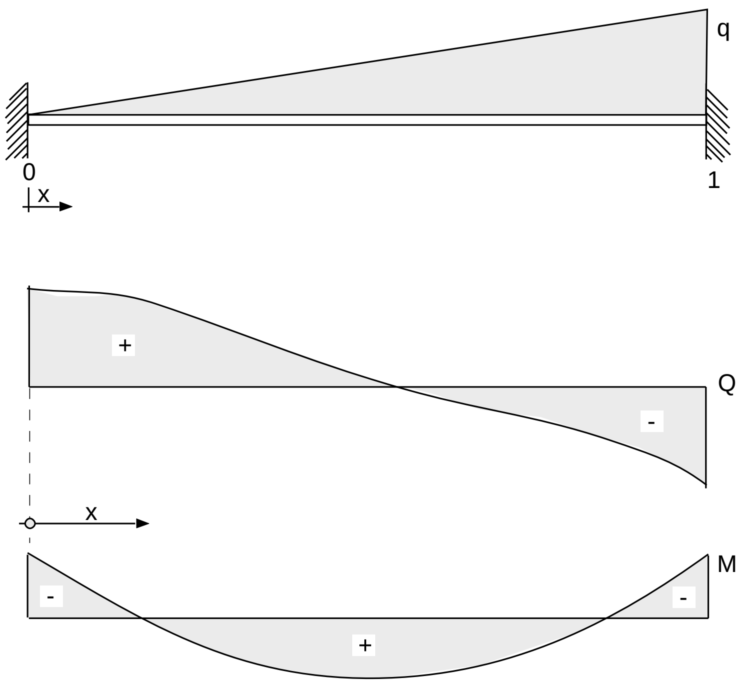
$$Q\,(x) = \frac{qL}{20}\left(3 - 10\,\frac{x^2}{L^2}\right)$$

$$M\,(x) = -\,\frac{qL^2}{60}\left(-\,2 + 9\,\frac{x}{L} - 10\,\frac{x^3}{L^3}\right)$$

$$+M_{max} = \frac{qL^2}{46.6}\ \text{at}\ x = 0.548\ L$$

$$-M_{max} = M(L) = -\,\frac{qL^2}{20}$$

$$M = 0\ \text{at}\ x = 0.237\ L\ \text{and}\ 0.808\ L$$



$$w_1\,(x) = \frac{qxL^3}{120D}\left(\frac{2x}{L} - \frac{3x^2}{L2} + \frac{x^4}{L^4}\right)$$

$$w_2\,(x) = \frac{qL}{20V}\left(3x - \frac{10}{3}\,\frac{x^3}{L^2}\right)$$

a	Length of panel
b	Width of beam/panel
c	Core thickness
d	Distance between centrelines of opposite faces
e	Width of core
e_i	Tensile strains
g	$1-u_x u_y$
h	Overall thickness of beam/panel
m,n	Suffixes denoting mode of deformation m; half-vawes in x-direction n; half-vawes in y-direction
q	Distributed load (panels: uniform pressure)
t	Face thickness
v	Used as a temporary variable in definition of equations
w	Deflection
x, y, z	Rectangular coordinates
A	Area of cross-section
A_i	Cross-section coefficients for strain energies
$B_{i,ij}$	Modified shear stiffnesses
C_i	Constants in derivation of equations
D	Flexural rigidity
E	Moduli of elasticity
G	Moduli of rigidity (shear moduli)
H	Potential energy
I	Second moment of area
K	Buckling coefficient
L	Span
M_i	Bending moment
M_{ij}	Twisting moment
N	Edge load for panels
$N_{i,ij}$	Membrane forces
P	Axial load
Q	Transverse force
S	First moment of area
U	Strain energy
V	Shear stiffness
V_i	Potential energies
W	Point load
α	Factor in buckling equations
β	Constants for different Euler cases
β_ι	Coefficients for calculating stresses and deflection in a sandwich panel
γ	Shear strain
κ	Curvature
λ	Coefficient for magnitude of angle of deflection
μ	Coefficient for magnitude of angle of deflection
ν	Poisson´s ratio
ρ	Stiffness constant for panel
σ	Tensile/compressive stress
τ	Shear stress
ϑ	Used in angle definitions in derivation of equations
Δ	Displacement at a certain point
Ω	Quantity for panel bending equations

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